1. INTRODUCTION

The first approaches to the understanding of human emotions were made by C. Darwin and I. Pavlov. Darwin [1] explained the main expressive movements in man by three principles: the principle of service, the principle of opposite action, and independence of habit. Pavlov in 1932 suggested that emotions could be viewed as lack of fit (mismatch) between the ready-made inner stereotype prepared in advance by the brain and the changing external circumstances [2]. Simonov concluded in 1964 that "emotion is a reflection by the human brain... of some actual need (its quality and magnitude) and the probability (possibility) of satisfying this need, which the brain estimates on the basis of genetic and previously acquired individual experience" [3]. This idea served as the basis for Simonov’s need/information theory, in which the formation of emotions E was represented by the structural formula $E = f(\Pi, U_H - U_C)$, where $\Pi$ is the strength and quality of the actual need, $U_H - U_C$ is an estimate of the probability of satisfying the need. McGrass’s formula [4] for the degree of stress $S = C_0(D - C)$ has a similar structure: here $C_0$ is the degree of importance of the demand, $D$ is the perceived demand, $C$ is the perceived possibility. We should also mention Spence’s equation for the response (emotion) potential, introduced by Hall: it depends on the strength of habit $H$ and the strength of the emotion level $E = HD$. To the best of our knowledge, no quantitative computations using these formulas have been attempted so far.

In this paper, we apply a model of information processing by memory to describe human adaptation to emotional factors. This approach has been previously applied to the study of adaptation in analyzer systems [5].

2. ANALYSIS OF EXPERIMENTAL DATA

A limited number of psychological experiments have attempted to quantify the effect of emotional factors on human activity. Of the greatest interest are the experiments of Mileryan [6], who has studied the psychological aspects of skill acquisition by operators using a flight simulator.

Mileryan’s experiments compared various criteria characterizing the efficiency of performance of sensomotor functions under extreme conditions with the same criteria under optimal working conditions. Extreme conditions were produced by the introduction of stress factors, which included 1) painful electric current shocks, which were automatically applied whenever the subject exceeded the programmed bounds, 2) strict time limits on the performance of tasks (delay of the subject’s next action beyond a certain limit triggered an obstruction that covered the console and thus threatened the completion of the entire task).

Analysis of Mileryan’s experiments has shown that training curves may provide a criterion for the effect of emotions. A feature of training curves under emotional conditions is improvement of performance in the initial period of skill formation, followed by deterioration in the final stage. Mileryan claims that this effect has been observed only for one group of subjects. His conclusion may be explained by the absence of theoretical learning curves to which the experimental results could be compared. If Mileryan’s experiments are superimposed on the theoretical results calculated from the formulas of [7], then we see (Figs. 1-4) that the emotional effect is indeed observed for all groups of subjects. Thus, emotional stability can be quantitatively determined from training curves. The main criterion of emotional stability is the speed of adaptation to emotional factors, which arise when man encounters a new activity that requires overcoming certain difficulties. As a result of adaptation,
Fig. 1. Formation of a sensomotor skill in a subject from the emotionally most stable (first) group: 1 and 2 are Mileryan’s experiments under optimal and extreme conditions, respectively; 3 are calculations from formula (1) for $\tau_1 = 15.1$; 4 are calculations by formula (6); numbered experimental points were obtained for different levels of electric skin irritants: 11-13 for 30 mV, 5-6 for 60 mV, 7-9 for 100 mV.

Fig. 2. Formation of a sensomotor skill in a subject of group 3 (emotionally less stable than subjects of group 1): 1 and 2 are Mileryan’s experiments under optimal and extreme conditions, respectively; 3 are calculations from formula (1) for $\tau_1 = 45$; 4 are calculations by formula (6); 5, 6, 14 for 50 mV, 7-10 for 100 mV, 11-13 for 25 mV.

Fig. 3. Formation of a sensomotor skill in a subject between groups 1 and 2: 1 and 2 are Mileryan’s experiments under optimal and extreme conditions, respectively; 3 are calculations from formula (1) for $\tau_1 = 20$; 4 are calculations by formula (6); 5, 6, 14 for 50 mV, 7-9 for 75 mV, 11-13 for 25 mV.
Fig. 4. Formation of a sensomotor skill in a subject intermediate between groups 2 and 3: 1 are Mileryan's experiments under optimal conditions, respectively; 2 are calculations from formula (1) for \( \tau_1 = 31 \); 3 are calculations by formula (6); 4 for 15 mV, 5-13 for 25 mV, 6 for 8 mV, 9-10 for 45 mV, 11-13 for 10 mV.

Fig. 5. Adaptation to emotional factors in the process of acquisition of operator skills: based on Mileryan's experiments and calculations from (5) for \( \tau_1 = 45 \) (1), \( \tau_1 = 31 \) (2), \( \tau_1 = 20 \) (3), and \( \tau_1 = 15.1 \) (4). The remaining legend as in Figs. 1-4.

dynamic stereotypes begin to be generated automatically in the process of training and the emotional state crosses the zero level, reversing the sign of the effect.

3. A MODEL OF ADAPTATION TO EMOTIONAL EFFECTS

We represent the effect of an emotional factor by a deviation (with an appropriate sign) from the level achieved in the process of normal skill acquisition. Given the description of the learning curve in multiply repeated training [7]

\[
\bar{U}_n = 1 - (1 - \bar{U}_1)^n,
\]

which has been obtained from the equation of conservation of the information flow

\[
\frac{d\Delta U_n}{dt} = (1 - \bar{U}_{n-1})G - \frac{\Delta U_n}{T},
\]

we can calculate the learning curve from (1) and find the adaptive dependence of the operator on emotional factors. Here \( \Delta U_n = U_n - U_{n-1} \) is the increase of information in human memory during the \( n \)-th repetition, \( \bar{U}_n = U_n/U_\Sigma \), \( T \) is the time constant of information processing by human memory, \( G \) is the rate of inflow of information to memory, \( U_\Sigma \) is the total amount of information acquired by learning,

\[
\bar{U}_1 = (1 - e^{-\tau_1})/\overline{\tau_1}; \ \overline{\tau}_1 = \tau_1/T; \ \tau_1 = U_\Sigma / G.
\]
Let $\Delta Q_n = Q_n - U_n$ be the change in the amount of information acquired by memory due to the emotional effect. Then Mileryan's experimental results can be represented in the form shown in Fig. 5. Note the fluctuations in the amount of information acquired by repetition (this will be discussed later). In what follows, we consider the average $\Delta Q_n = \Delta Q_n/U_n$, which is represented similarly to the adaptive dependence previously established for analyzer systems [5]:

$$\Delta Q_n = \bar{Q}_m + \bar{Q}_0 \exp(-\theta n),$$  

where $\bar{Q}_m = -0.1$, $\bar{Q}_0 = 0.4$.

The constant $\theta$ is determined from the following considerations. Analysis of experimental data easily shows that the inversion point (the point when the effect of emotions on learning efficiency changes its sign) approximately corresponds to $n_0 = n_T/2$, where $n_T$ is the number of repetitions to complete learning of the material [7]. For our purposes, we assume that $n_T$ corresponds to learning level $\bar{U}_n = 0.95$. Then from dependence (1) and its approximation [7]

$$U_n^* = 1 - \xi \exp(-\xi n_T/\bar{\tau}_1) = 0.95$$

we can easily find $n_T = 3\bar{\tau}_1$. Here $\xi$ is a known coefficient, which is close to 1 for $\bar{\tau}_1 > 8$. Hence we obtain $n_0 = 3\bar{\tau}_1/2$. Since $\Delta Q_n = 0$ for $n = n_0$, we equate (3) to zero for this $n$ and obtain the constant $\theta = 1/\bar{\tau}_1$. Thus, (3) is finally written in the form

$$\Delta Q_n = \bar{Q}_0 e^{-n/\bar{\tau}_1} + \bar{Q}_m.$$  

If we allow for the obvious dependence between the elapsed time $\tau$ and the number of repetitions $n = \tau/\bar{\tau}_1$, then we can obtain the dependence $\Delta Q_n(\tau)$ from (5). Since we have assumed that emotions affect linearly the attained level of learning, the emotional effect $\bar{Q}_n = \bar{U}_n + \Delta \bar{Q}_n$ in these cases takes the final form

$$\bar{Q}_n = 1 - \left(1 - \frac{1 - e^{-\xi}}{\bar{\tau}_1}\right)^n + \bar{Q}_0 e^{-n/\bar{\tau}_1} + \bar{Q}_m.$$  

4. ANALYSIS OF CALCULATIONS

Comparison of calculations by formula (6) with Mileryan's experiments shows a good fit between the two sets of data (see dashed curves in Figs. 1-4). There is a relationship between the speed of formation of a dynamic stereotype and emotional stability of the subject: the greater the number of repetitions required to produce a sensomotor skill (i.e., the higher the parameter $\bar{\tau}_1$), the more unstable is the subject to emotional effects.

A quantitative criterion characterizing individual learning is the time constant of information processing in human memory $T$. The speed of skill formation, as we see from (1), depends on the length of one repetition of all the material being learned (in a relative expression) $\bar{\tau}_1 = U_2/GT$. Since neither the total information $U_2$ for learning and repetition associated with operator activity of the form considered by Mileryan nor the rate of information inflow to memory $G$ is known, we may reasonably assume to first approximation that the time for one repetition $\tau = U_2/G$ is the same in all experiments. (This was not specially stipulated, but apparently on average this was in fact so.) Then, given the parameter $\bar{\tau}_1$ for each subject, we can find the ratio of the time constants $T$ for different groups of people (Mileryan divided the subjects into three groups [6]).

We see from the data in Figs. 1-4 that the emotionally most stable individuals are characterized by the parameter value $\bar{\tau}_1 = 15.1$; individuals between groups 1 and 2 are characterized by $\bar{\tau}_1 = 20$; and individuals between groups 2 and 3 by $\bar{\tau}_1 = 31$. The least stable subjects had $\bar{\tau}_1 = 45$. Thus, if emotions are measured in units of the time constant $T = \bar{T}_0$ of the emotionally most stable people, a quantitative characteristic of emotional stability is provided by the ratio $\kappa = T/\bar{T}_0 = \bar{\tau}_1/\bar{\tau}_1$. Hence we conclude that the time constant $\bar{T}_0$ determined from learning curves can be used as a measure of emotions. If $\kappa = 1$ for the emotionally most stable operators, then for emotionally less stable individuals on the border of groups 1, 2 and 2, 3 in Mileryan's classification $\kappa$ is respectively $3/4$ and $1/2$. For the emotionally least stable individuals, $\kappa = 1/3$. Of course we should stress that these results were obtained using a first-approximation model based, in a certain sense, on a particular
5. THE CASE OF COMPLEX EMOTIONS

Let us now consider an interaction model of complex emotions (identified by numbers in Figs. 1-4), which in our case are produced by the combined action of electric skin irritants of different intensity and the time limit on the performance of a task, with a threat of interruption by a special moving obstruction. We see from Figs. 1-4 that the application of emotiogenic factors of this kind is accompanied by a reduction of the attained skill level compared to previous experiments (conducted under optimal working conditions).

A mathematical model of adaptation to complex emotions is constructed from the following considerations. We assume that emotional effects follow an adaptive rule, and we consider separately the threat of premature termination of the task by the closing obstruction, which is posed before the beginning of the experiment. This type of adaptation is similar to that described in (5). Adaptation to electric shocks requires a somewhat different approach. This is clearly an impulsive effect, after which the trace gradually decays until a new shock is applied. After a new and stronger current shock, the count starts from the level that produced a new irritation (initial condition). Mathematically this can be described in the same way as in the theory
of information processing by human memory [7]: the reduction in the rate of information acquisition during repeated training with impulsive application of an emotional irritant is described by the equation

\[
\frac{d\Delta E}{dt} = -\frac{b}{T} \Delta E, \tag{7}
\]

where \( \Delta E = E - \xi E_0 \), where \( \xi E_0 \) is the residual emotional information from the previous irritant remaining at the time when the next irritant is applied, \( \xi \) is a correction coefficient that allows for the magnitude of the initial emotion (see below).

Integrating from 0 to \( \tau \) and from \( \Delta E_0 \) to \( \Delta E \),

\[
\Delta E = \Delta E_0 \exp (- \frac{b \tau}{T}), \tag{8}
\]

or passing to absolute values (instead of increments), we obtain allowing for the dependence between \( n \) and \( \tau \)

\[
E = \xi E_0 + \Delta E_0 \exp (-\frac{b n \tau}{T_1}). \tag{9}
\]

The coefficient \( \xi \) according to our estimates can be approximately taken equal to \( \xi = 1/\bar{g} \), where \( \bar{g} = g/g_0 \) is the relative level of electric skin irritation, \( g_0 \) is the initial irritant. The coefficient \( b \sim 1/T_1^2 \), and we can thus pass to the same exponent \( n/T_1 \) as in (6). The proportionality coefficients was taken equal to 10 from empirical considerations \( (b = 10/T_1^2) \). Thus, (9) takes the final form (expressed in relation to \( U_n \))

\[
E = \frac{\bar{E}_0}{\bar{g}} + \Delta \bar{E}_0 \exp (-10n/T_1) \tag{10}
\]

From experimental data, \( \Delta \bar{E}_0 = 0.7 \).

While emotions of the first type — threat — initially have a positive effect (improving information acquisition), emotions of the second type — electric shock — always have a negative effect. Therefore, the total change of emotions is described by the formula

\[
\Delta \bar{A} = \Delta \bar{Q}_n - \bar{E} = \bar{Q}_0 e^{-n/T_1} + \bar{E}_0/\bar{g} + \Delta \bar{E}_0 e^{-10n/T_1}, \tag{11}
\]

or, allowing for the numerical values,

\[
\Delta \bar{A} = 0.4 e^{-n/T_1} + \bar{E}_0/\bar{g} + 0.7 e^{-10n/T_1} - 0.1 \tag{12}
\]

Note that if the electric irritation strength is subsequently reduced, the negative emotion is determined by the previous trace produced by the stronger irritant (this case was observed in Mileryan’s experiments shown in Fig. 6c). Comparison of calculation results from formula (12) with Mileryan’s experiments (see dashed curves in Fig. 6) on the whole reveals both quantitative and qualitative fit. This confirms the applicability of the proposed approach to quantitative description of the effect of emotions on man.

6. MODEL OF EMOTIONAL EFFECTS WITH FLUCTUATING COMPONENT

The simple approach considered in the previous sections describes the formation of skills in the presence of emotions in average terms, without allowing for the experimentally observed fluctuations of the process. The change in information inflow to human memory due to the emotional effect \( \Delta Q \) can be determined by solving an equation similar to the conservation equation of information flow (2) in the form

\[
\frac{d\Delta Q}{dt} = G - \frac{\Delta Q}{\theta}, \tag{13}
\]

where \( \theta \) is the time constant of information processing by human memory.
Fig. 7. Comparison of experimental data (1) with calculations from formula (18) for \( \gamma_1 = 15.1 \) and \( \epsilon = 120 \) (2) and \( \epsilon = 0.6 \) (3).

If we view emotions as inertia associated with given circumstances, then the rate of inflow of emotional information \( G' \) can be represented to first approximation in the form

\[
\frac{dG'}{dt} = -\frac{\Delta Q}{\theta},
\]

where \( \theta \) is a constant. The system of these two equations can be reduced to a single equation

\[
\frac{d^2\Delta Q}{dt^2} + \frac{1}{\theta} \frac{d\Delta Q}{dt} + \frac{\Delta Q}{\theta} = 0,
\]

whose solution is

\[
\Delta Q = C_1 \exp \left( -\frac{\tau}{2\theta} \right) \sin \left[ 0.5\lambda (\tau - C_2) \right],
\]

where \( \lambda^2 = \frac{4}{\theta} - 1/\theta^2 > 0 \).

Let \( \gamma = \tau/T \), where \( T \) is the time constant of information processing in the absence of emotional effects. Then denoting the ratio of time constants by \( \beta = T/\theta \), we obtain the relationship \( \tau/\theta = \beta \gamma \). The integration constants are determined from the following conditions: a) for \( \tau = 0 \), \( \Delta Q = \Delta Q_0 \); b) for \( \tau = \gamma_0 = n_0 \gamma_1 = 1.5 \gamma_1^2 \), \( \Delta Q = 0 \) (inversion point). The second condition may be taken in limit form: as \( n \rightarrow n_0 = 3\gamma_1 \), \( \Delta Q \rightarrow \Delta Q_\alpha \) (= -0.1). The final results are quantitatively fairly close.

Omitting the intermediate steps and using our notation, we obtain the solution of Eq. (15) in the form

\[
\Delta Q = \Delta Q_0 \exp \left( -0.5\beta n\gamma_1 \right) \sin \left[ 0.5\beta \sqrt{4\epsilon - 1} (1.5\gamma_1^2 - n\gamma_1 - 2\pi m/\sqrt{4\epsilon - 1}) \right] \sin \left[ 0.5\beta \sqrt{4\epsilon - 1} (1.5\gamma_1^2 - 4\pi m/\beta \sqrt{4\epsilon - 1}) \right],
\]

where \( \epsilon = \theta^2/\beta, \ m = 0,1,2, \ldots \).

An interesting particular case is when the exponents in formulas (5) and (17) are close to one another, which gives the relationship \( \beta = 2/\gamma_1 \), that simplifies the final expression. We consider only the first harmonic, i.e., \( m = 0 \). Then (17) takes the form (for \( \nu = 0.5\sqrt{4\epsilon - 1} \beta \gamma_1^2 \))

\[
\Delta Q = \Delta Q_0 \exp \left( -n/\gamma_1 \right) \sin \left[ \nu(1.5 - n/\gamma_1) \right] \sin \left( 1.5\nu \right).
\]
The proposed model is different from that considered in Sec. 3 because it explicitly allows for the fluctuating component, which is typically observed during skill formation under emotional stress. Calculations using the final formula (18) were compared with the results of Mileryan’s experiments. Since the ratio $\varepsilon$ cannot be determined in advance, we treat it as a parameter, whose value can be estimated by matching the calculations with the experiments. Such calculations were carried out for a wide range of $\varepsilon$. For small $\varepsilon$ (less than 1) we obtained the previously considered averaged solution, which adequately describes adaptation to emotions without fluctuation. In this case, the value of $\varepsilon$ ensuring consistency with experimental results depends on the parameter $\tau_1$: $\varepsilon = 0.83 - 0.012\tau_1$. Fluctuating components appear for large $\varepsilon$: to first approximation, a satisfactory fit is achieved for $\varepsilon = 126$ (with the exception of the initial stage, when the fluctuation amplitude is large). A specimen graph comparing the calculations from (18) with experimental results is shown in Fig. 7. Thus, our fairly simple model provides a good qualitative and quantitative fit with the experimental findings.

CONCLUSIONS

1. The results of this paper establish the possibility of successful mathematical modeling of the highly complex phenomenon representing human emotions.

2. The dependences obtained in this study describe with a fairly good fit human adaptation to various emotiogenic irritants.

3. The development of the proposed approach based on the model of information processing by human memory previously proposed by the authors is promising for application in other important cases.

REFERENCES

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