I.12 REMARKS ON THE MIND-BODY QUESTION

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Introductory Comments

F. Dyson, in a very thoughtful article,[1] points to the everbroadening scope of scientific inquiry. Whether or not the relation of mind to body will enter the realm of scientific inquiry in the near future—and the present writer is prepared to admit that this is an open question—it seems worthwhile to summarize the views to which a dispassionate contemplation of the most obvious facts leads. The present writer has no other qualification to offer his views than has any other physicist and he believes that most of his colleagues would present similar opinions on the subject, if pressed.

Until not many years ago, the "existence" of a mind or soul would have been passionately denied by most physical scientists. The brilliant successes of mechanistic and, more generally, macroscopic physics and of chemistry overshadowed the obvious fact that thoughts, desires, and emotions are not made of matter, and it was nearly universally accepted among physical scientists that there is nothing besides matter. The epitome of this belief was the conviction that, if we knew the positions and velocities of all atoms at one instant of time, we could compute the fate of the universe for all future. Even today, there are adherents to this

[1] F. J. Dyson, *Scientific American,* 199, 74 (1958). Several cases are related in this article in which regions of inquiry, which were long considered to be outside the province of science, were drawn into this province and, in fact, became focuses of attention. The best-known example is the interior of the atom, which was considered to be a metaphysical subject before Rutherford's proposal of his nuclear model, in 1911.

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view[2] though fewer among the physicists than – ironically enough – among biochemists.

There are several reasons for the return, on the part of most physical scientists, to the spirit of Descartes's "*Cogito ergo sum*," which recognizes the thought, that is, the mind, as primary. First, the brilliant successes of mechanics not only faded into the past; they were also recognised as partial successes, relating to a narrow range of phenomena, all in the macroscopic domain. When the province of physical theory was extended to encompass microscopic phenomena, through the creation of quantum mechanics, the concept of consciousness came to the fore again: it was not possible to formulate the laws of quantum mechanics in a fully consistent way without reference to the consciousness.[3] All that quantum mechanics purports to provide are probability connections between subsequent impressions (also called "apperceptions") of the consciousness, and even though the dividing line between the observer, whose consciousness is being affected, and the observed physical object can be shifted towards the one or the other to a considerable degree,[4] it cannot be eliminated. It may be premature to believe that the present philosophy of quantum mechanics will remain a permanent feature of future physical theories; it will remain remarkable, in whatever way our future concepts may develop, that the very study of the external world led to the conclusion that the content of the consciousness is an ultimate reality.

It is perhaps important to point out at this juncture that the question concerning the existence of almost anything (even the whole external world) is not a very relevant question. All of us recognize at once how meaningless the query concerning the existence of the electric field in vacuum would be. All that is relevant is that the concept of the electric

[2] The book most commonly blamed for this view is E. F. Haeckel's *Welträtsel* (1899). However, the views propounded in this book are less extreme (though more confused) than those of the usual materialistic philosophy.

[3] W. Heisenberg expressed this most poignantly [*Daedalus,* 87, 99 (1958)]: "The laws of nature which we formulate mathematically in quantum theory deal no longer with the particles themselves but with our knowledge of the elementary particles." And later: "The conception of objective reality . . . evaporated into the . . . mathematics that represents no longer the behavior of elementary particles but rather our knowledge of this behavior." The "our" in this sentence refers to the observer who plays a singular role in the epistemology of quantum mechanics. He will be referred to in the first person and statements made in the first person will always refer to the observer.

[4] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik* (Berlin: Julius Springer, 1932), Chapter VI; English translation (Princeton, N.J.: Princeton University Press, 1955).

field is useful for communicating our ideas and for our own thinking. The statement that it "exists" means only that: (*a*) it can be measured, hence uniquely defined, and (*b*) that its knowledge is useful for understanding past phenomena and in helping to foresee further events. It can be made part of the *Weltbild.* This observation may well be kept in mind during the ensuing discussion of the quantum mechanical description of the external world.

The Language of Quantum Mechanics

The present and the next sections try to describe the concepts in terms of which quantum mechanics teaches us to store and communicate information, to describe the regularities found in nature. These concepts may be called the language of quantum mechanics. We shall not be interested in the regularities themselves, that is, the contents of the book of quantum mechanics, only in the language. It may be that the following description of the language will prove too brief and too abstract for those who are unfamiliar with the subject, and too tedious for those who are familiar with it.[5] It should, nevertheless, be helpful. However, the knowledge of the present and of the succeeding section is not necessary for following the later ones, except for parts of the section on the Simplest Answer to the Mind-Body Question.

Given any object, all the possible knowledge concerning that object can be given as its wave function. This is a mathematical concept the exact nature of which need not concern us here—it is composed of a (countable) infinity of numbers. If one knows these numbers, one can foresee the behavior of the object as far as it *can* be foreseen. More precisely, the wave function permits one to foretell with what probabilities the object will make one or another impression on us if we let it interact with us either directly, or indirectly. The object may be a radiation field, and its wave function will tell us with what probability we shall see a

[5] The contents of this section should be part of the standard material in courses on quantum mechanics. They are given here because it may be helpful to recall them even on the part of those who were at one time already familiar with them, because it is not expected that every reader of these lines had the benefit of a course in quantum mechanics, and because the writer is well aware of the fact that most courses in quantum mechanics do not take up the subject here discussed. See also, in addition to references 3 and 4, W. Pauli, *Handbuch der Physik,* Section 2.9, particularly page 148 (Berlin: Julius Springer, 1933). Also F. London and E. Bauer, *La Théorie de l'observation en mécanique quantique* (Paris: Hermann and Co., 1939). The last authors observe (page 41), "Remarquons le rôle essential que joue la conscience de l'observateur. . . ."

flash if we put our eyes at certain points, with what probability it will leave a dark spot on a photographic plate if this is placed at certain positions. In many cases the probability for one definite sensation will be so high that it amounts to a certainty—this is always so if classical mechanics provides a close enough approximation to the quantum laws.

The information given by the wave function is communicable. If someone else somehow determines the wave function of a system, he can tell me about it and, according to the theory, the probabilities for the possible different impressions (or "sensations") will be equally large, no matter whether he or I interact with the system in a given fashion. In this sense, the wave function "exists."

It has been mentioned before that even the complete knowledge of the wave function does not permit one always to foresee with certainty the sensations one may receive by interacting with a system. In some cases, one event (seeing a flash) is just as likely as another (not seeing a flash). However, in most cases the impression (e.g., the knowledge of having or not having seen a flash) obtained in this way permits one to foresee later impressions with an increased certainty. Thus, one may be sure that, if one does not see a flash if one looks in one direction, one surely does see a flash if one subsequently looks in another direction. The property of observations to increase our ability for foreseeing the future follows from the fact that all knowledge of wave functions is based, in the last analysis, on the "impressions" we receive. In fact, the wave function is only a suitable language for describing the body of knowledge—gained by observations—which is relevant for predicting the future behaviour of the system. For this reason, the interactions which may create one or another sensation in us are also called observations, or measurements. One realises that *all* the information which the laws of physics provide consists of probability connections between subsequent impressions that a system makes on one if one interacts with it repeatedly, i.e., if one makes repeated measurements on it. The wave function is a convenient summary of that part of the past impressions which remains relevant for the probabilities of receiving the different possible impressions when interacting with the system at later times.

An Example

It may be worthwhile to illustrate the point of the preceding section on a schematic example. Suppose that all our interactions with the system consist in looking at a certain point in a certain direction at times

$t_0, t_0 + 1, t_0 + 2, \cdots$, and our possible sensations are seeing or not seeing a flash. The relevant law of nature could then be of the form: "If you see a flash at time t, you will see a flash at time $t + 1$ with a probability $\frac{1}{4}$, no flash with a probability $\frac{3}{4}$; if you see no flash, then the next observation will give a flash with the probability $\frac{3}{4}$, no flash with a probability $\frac{1}{4}$; there are no further probability connections." Clearly, this law can be verified or refuted with arbitrary accuracy by a sufficiently long series of observations. The wave function in such a case depends only on the last observation and may be ψ_1 if a flash has been seen at the last interaction, ψ_2 if no flash was noted. In the former case, that is for ψ_1, a calculation of the probabilities of flash and no flash after unit time interval gives the values $\frac{1}{4}$ and $\frac{3}{4}$; for ψ_2 these probabilities must turn out to be $\frac{3}{4}$ and $\frac{1}{4}$. This agreement of the predictions of the law in quotation marks with the law obtained through the use of the wave function is not surprising. One can either say that the wave function was invented to yield the proper probabilities, or that the law given in quotation marks has been obtained by having carried out a calculation with the wave functions, the use of which we have learned from Schrödinger.

The communicability of the information means, in the present example, that if someone else looks at time t, and tells us whether he saw a flash, we can look at time $t + 1$ and observe a flash with the same probabilities as if we had seen or not seen the flash at time t ourselves. In other words, he can tell us what the wave function is: ψ_1 if he did, ψ_2 if he did not see a flash.

The preceding example is a very simple one. In general, there are many types of interactions into which one can enter with the system, leading to different types of observations or measurements. Also, the probabilities of the various possible impressions gained at the next interaction may depend not only on the last, but on the results of many prior observations. The important point is that the impression which one gains at an interaction may, and in general does, modify the probabilities with which one gains the various possible impressions at later interactions. In other words, the impression which one gains at an interaction, called also *the result of an observation*, modifies the wave function of the system. The modified wave function is, furthermore, in general unpredictable before the impression gained at the interaction has entered our consciousness: it is the entering of an impression into our consciousness which alters the wave function because it modifies our appraisal of the probabilities for different impressions which we expect to receive in the future. It is at this point that the consciousness enters the theory unavoidably and unalterably. If one speaks in terms of the wave function, its changes are coupled with the entering of impressions into our consciousness. If one formulates the laws of quantum mechanics in terms of probabilities of impressions, these are *ipso facto* the primary concepts with which one deals.

It is natural to inquire about the situation if one does not make the observation oneself but lets someone else carry it out. What is the wave function if my friend looked at the place where the flash might show at time t? The answer is that the information available about the *object* cannot be described by a wave function. One could attribute a wave function to the joint system: friend plus object, and this joint system would have a wave function also after the interaction, that is, after my friend has looked. I can then enter into interaction with this joint system by asking my friend whether he saw a flash. If his answer gives me the impression that he did, the joint wave function of friend + object will change into one in which they even have separate wave functions (the total wave function is a product) and the wave function of the object is ψ_1. If he says no, the wave function of the object is ψ_2, i.e., the object behaves from then on as if I had observed it and had seen no flash. However, even in this case, in which the observation was carried out by someone else, the typical change in the wave function occurred only when some information (the *yes* or *no* of my friend) entered *my* consciousness. It follows that the quantum description of objects is influenced by impressions entering my consciousness.[6] Solipsism may be logically consistent with present quantum mechanics, monism in the sense of materialism is not. The case against solipsism was given at the end of the first section.

The Reasons for Materialism

The principal argument against materialism is not that illustrated in the last two sections: that it is incompatible with quantum theory. The

[6] The essential point is not that the states of objects cannot be described by means of position and momentum co-ordinates (because of the uncertainty principle). The point is, rather, that the valid description, by means of the wave function, is influenced by impressions entering our consciousness. See in this connection the remark of London and Bauer, quoted above, and S. Watanabe's article in *Louis de Broglie, Physicien et Penseur* (Paris: Albin Michel, 1952), p. 385.

principal argument is that thought processes and consciousness are the primary concepts, that our knowledge of the external world is the content of our consciousness and that the consciousness, therefore, cannot be denied. On the contrary, logically, the external world could be denied—though it is not very practical to do so. In the words of Niels Bohr,[7] "The word consciousness, applied to ourselves as well as to others, is indispensable when dealing with the human situation." In view of all this, one may well wonder how materialism, the doctrine[8] that "life could be explained by sophisticated combinations of physical and chemical laws," could so long be accepted by the majority of scientists.

The reason is probably that it is an emotional necessity to exalt the problem to which one wants to devote a lifetime. If one admitted anything like the statement that the laws we study in physics and chemistry are limiting laws, similar to the laws of mechanics which exclude the consideration of electric phenomena, or the laws of macroscopic physics which exclude the consideration of "atoms," we could not devote ourselves to our study as wholeheartedly as we have to in order to recognise any new regularity in nature. The regularity which we are trying to track down must appear as the all-important regularity—if we are to pursue it with sufficient devotion to be successful. Atoms were also considered to be an unnecessary figment before macroscopic physics was essentially complete—and one can well imagine a master, even a great master, of mechanics to say: "Light may exist but I do not need it in order to explain the phenomena in which I am interested." The present biologist uses the same words about mind and consciousness; he uses them as an expression of his disbelief in these concepts. Philosophers do not need these illusions and show much more clarity on the subject. The same is true of most truly great natural scientists, at least in their years of maturity. It is now true of almost all physicists—possibly, but not surely, because of the lesson we learned from quantum mechanics. It is also possible that we learned that the principal problem is no longer the fight with the adversities of nature but the difficulty of understanding ourselves if we want to survive.

[7] N. Bohr, *Atomic Physics and Human Knowledge,* section on "Atoms and Human Knowledge," in particular p. 92 (New York: John Wiley & Sons, 1960).

[8] The quotation is from William S. Beck, *The Riddle of Life, Essay in Adventures of the Mind* (New York: Alfred A. Knopf, 1960), p. 35. This article is an eloquent statement of the attitude of the open-minded biologists toward the questions discussed in the present note.

Simplest Answer to the Mind-Body Question

Let us first specify the question which is outside the province of physics and chemistry but is an obviously meaningful (because operationally defined) question: Given the most complete description of my body (admitting that the concepts used in this description change as physics develops), what are my sensations? Or, perhaps, with what probability will I have one of the several possible sensations? This is clearly a valid and important question which refers to a concept—sensations—which does not exist in present-day physics or chemistry. Whether the question will eventually become a problem of physics or psychology, or another science, will depend on the development of these disciplines.

Naturally, I have direct knowledge only of my own sensations and there is no strict logical reason to believe that others have similar experiences. However, everybody believes that the phenomenon of sensations is widely shared by organisms which we consider to be living. It is very likely that, if certain physico-chemical conditions are satisfied, a consciousness, that is, the property of having sensations, arises. This statement will be referred to as our first thesis. The sensations will be simple and undifferentiated if the physico-chemical substrate is simple; it will have the miraculous variety and colour which the poets try to describe if the substrate is as complex and well organized as a human body.

The physico-chemical conditions and properties of the substrate not only create the consciousness, they also influence its sensations most profoundly. Does, conversely, the consciousness influence the physico-chemical conditions? In other words, does the human body deviate from the laws of physics, as gleaned from the study of inanimate nature? The traditional answer to this question is, "No": the body influences the mind but the mind does not influence the body.[9] Yet at least two reasons can be given to support the opposite thesis, which will be referred to as the second thesis.

The first and, to this writer, less cogent reason is founded on the

[9] This writer does not profess to a knowledge of all, or even of the majority of all, metaphysical theories. It may be significant, nevertheless, that he never found an affirmative answer to the query of the text—not even after having perused the relevant articles in the earlier (more thorough) editions of the *Encyclopaedia Britannica.*

quantum theory of measurements, described earlier in sections 2 and 3. In order to present this argument, it is necessary to follow my description of the observation of a "friend" in somewhat more detail than was done in the example discussed before. Let us assume again that the object has only two states, ψ_1 and ψ_2. If the state is, originally, ψ_1, the state of object plus observer will be, after the interaction, $\psi_1 \times \chi_1$; if the state of the object is ψ_2, the state of object plus observer will be $\psi_2 \times \chi_2$ after the interaction. The wave functions χ_1 and χ_2 give the state of the observer; in the first case he is in a state which responds to the question "Have you seen a flash?" with "Yes"; in the second state, with "No." There is nothing absurd in this so far.

Let us consider now an initial state of the object which is a linear combination $\alpha\,\psi_1 + \beta\,\psi_2$ of the two states ψ_1 and ψ_2. It then *follows* from the linear nature of the quantum mechanical equations of motion that the state of object plus observer is, after the interaction, $\alpha\,(\psi_1 \times \chi_1) + \beta\,(\psi_2 \times \chi_2)$. If I now ask the observer whether he saw a flash, he will with a probability $|\alpha|^2$ say that he did, and in this case the object will also give to me the responses as if it were in the state ψ_1. If the observer answers "No"—the probability for this is $|\beta|^2$—the object's responses from then on will correspond to a wave function ψ_2. The probability is zero that the observer will say "Yes," but the object gives the response which ψ_2 would give because the wave function $\alpha\,(\psi_1 \times \chi_1) + \beta\,(\psi_2 \times \chi_2)$ of the joint system has no $(\psi_2 \times \chi_1)$ component. Similarly, if the observer denies having seen a flash, the behavior of the object cannot correspond to χ_1 because the joint wave function has no $(\psi_1 \times \chi_2)$ component. All this is quite satisfactory: the theory of measurement, direct or indirect, is logically consistent so long as I maintain my privileged position as ultimate observer.

However, if after having completed the whole experiment I ask my friend, "What did you feel about the flash before I asked you?" he will answer, "I told you already, I did [did not] see a flash," as the case may be. In other words, the question whether he did or did not see the flash was already decided in his mind, before I asked him.[10] If we accept this, we are driven to the conclusion that the proper wave func-

[10] F. London and E. Bauer (*op. cit.*, reference 5) on page 42 say, "Il [l'observateur] dispose d'une faculté caractéristique et bien familière, que nous pouvons appeler la 'faculté d'introspection': il peut se rendre compte de manière immédiate de son propre état."

tion immediately after the interaction of friend and object was already either $\psi_1 \times \chi_1$ or $\psi_1 \times \chi_2$ and not the linear combination $\alpha (\psi_1 \times \chi_1) + \beta (\psi_2 \times \chi_2)$. This is a contradiction, because the state described by the wave function $\alpha (\psi_1 \times \chi_1) + \beta (\psi_2 \times \chi_2)$ describes a state that has properties which neither $\psi_1 \times \chi_1$ nor $\psi_2 \times \chi_2$ has. If we substitute for "friend" some simple physical apparatus, such as an atom which may or may not be excited by the light-flash, this difference has observable effects and *there is no doubt that* $\alpha (\psi_1 \times \chi_1) + \beta \cdot (\psi_2 \times \chi_2)$ *describes the properties of the joint system correctly, the assumption that the wave function is either* $\psi_1 \times \chi_1$ or $\psi_2 \times \chi_2$ *does not.* If the atom is replaced by a conscious being, the wave function $\alpha (\psi_1 \times \chi_1) + \beta (\psi_2 \times \chi_2)$ (which also follows from the linearity of the equations) appears absurd because it implies that my friend was in a state of suspended animation before he answered my question.[11]

It follows that the being with a consciousness must have a different role in quantum mechanics than the inanimate measuring device: the atom considered above. In particular, the quantum mechanical equations of motion cannot be linear if the preceding argument is accepted. This argument implies that "my friend" has the same types of impressions and sensations as I—in particular, that, after interacting with the object, he is not in that state of suspended animation which corresponds to the wave function $\alpha (\psi_1 \times \chi_1) + \beta (\psi_2 \times \chi_2)$. It is not necessary to see a contradiction here from the point of view of orthodox quantum mechanics, and there is none if we believe that the alternative is meaningless, whether my friend's consciousness contains either the impression of having seen a flash or of not having seen a flash. However, to deny the existence of the consciousness of a friend to this extent is surely an

[11] In an article which will appear soon [*Werner Heisenberg und die Physik unserer Zeit* (Braunschweig: Friedr. Vieweg, 1961)] G. Ludwig discusses the theory of measurements and arrives at the conclusion that quantum mechanical theory cannot have unlimited validity (see, in particular, Section IIIa, also Ve). This conclusion is in agreement with the point of view here represented. However, Ludwig believes that quantum mechanics is valid only in the limiting case of microscopic systems, whereas the view here represented assumes it to be valid for all inanimate objects. At present, there is no clear evidence that quantum mechanics becomes increasingly inaccurate as the size of the system increases, and the dividing line between microscopic and macroscopic systems is surely not very sharp. Thus, the human eye can perceive as few as three quanta, and the properties of macroscopic crystals are grossly affected by a single dislocation. For these reasons, the present writer prefers the point of view represented in the text even though he does not wish to deny the possibility that Ludwig's more narrow limitation of quantum mechanics may be justified ultimately.

unnatural attitude, approaching solipsism, and few people, in their hearts, will go along with it.

The preceding argument for the difference in the roles of inanimate observation tools and observers with a consciousness—hence for a violation of physical laws where consciousness plays a role—is entirely cogent so long as one accepts the tenets of orthodox quantum mechanics in all their consequences. Its weakness for providing a specific effect of the consciousness on matter lies in its total reliance on these tenets—a reliance which would be, on the basis of our experiences with the ephemeral nature of physical theories, difficult to justify fully.

The second argument to support the existence of an influence of the consciousness on the physical world is based on the observation that we do not know of any phenomenon in which one subject is influenced by another without exerting an influence thereupon. This appears convincing to this writer. It is true that under the usual conditions of experimental physics or biology, the influence of any consciousness is certainly very small. "We do not need the assumption that there is such an effect." It is good to recall, however, that the same may be said of the relation of light to mechanical objects. Mechanical objects influence light—otherwise we could not see them—but experiments to demonstrate the effect of light on the motion of mechanical bodies are difficult. It is unlikely that the effect would have been detected had theoretical considerations not suggested its existence, and its manifestation in the phenomenon of light pressure.

More Difficult Questions

Even if the two theses of the preceding section are accepted, very little is gained for science as we understand science: as a correlation of a body of phenomena. Actually, the two theses in question are more similar to existence theorems of mathematics than to methods of construction of solutions and we cannot help but feel somewhat helpless as we ask the much more difficult question: how could the two theses be verified experimentally? i.e., how could a body of phenomena be built around them. It seems that there is no solid guide to help in answering this question and one either has to admit to full ignorance or to engage in speculations.

Before turning to the question of the preceding paragraph, let us note in which way the consciousnesses are related to each other and to the physical world. The relations in question again show a remarkable similarity to the relation of light quanta to each other and to the material bodies with which mechanics deals. Light quanta do not influence each other directly[12] but only by influencing material bodies which then influence other light quanta. Even in this indirect way, their interaction is appreciable only under exceptional circumstances. Similarly, consciousnesses never seem to interact with each other directly but only via the physical world. Hence, any knowledge about the consciousness of another being must be mediated by the physical world.

At this point, however, the analogy stops. Light quanta can interact directly with virtually any material object but each consciousness is uniquely related to some physico-chemical structure through which alone it receives impressions. There is, apparently, a correlation between each consciousness and the physico-chemical structure of which it is a captive, which has no analogue in the inanimate world. Evidently, there are enormous gradations between consciousnesses, depending on the elaborate or primitive nature of the structure on which they can lean: the sets of impressions which an ant or a microscopic animal or a plant receives surely show much less variety than the sets of impressions which man can receive. However, we can, at present, at best, guess at these impressions. Even our knowledge of the consciousness of other men is derived only through analogy and some innate knowledge which is hardly extended to other species.

It follows that there are only two avenues through which experimentation can proceed to obtain information about our first thesis: observation of infants where we may be able to sense the progress of the awakening of consciousness, and by discovering phenomena postulated by the second thesis, in which the consciousness modifies the usual laws of physics. The first type of observation is constantly carried out by millions of families, but perhaps with too little purposefulness. Only very crude observations of the second type have been undertaken in the past, and all these antedate modern experimental methods. So far as it is known, all of them have been unsuccessful. However, every phenomenon is unexpected and most unlikely until it has been discovered—and some of them remain unreasonable for a long time after they have been discovered. Hence, lack of success in the past need not discourage.

[12] This statement is certainly true in an approximation which is much better than is necessary for our purposes.

Non-linearity of Equations as Indication of Life

The preceding section gave two proofs—they might better be called indications—for the second thesis, the effect of consciousness on physical phenomena. The first of these was directly connected with an actual process, the quantum mechanical observation, and indicated that the usual description of an indirect observation is probably incorrect if the primary observation is made by a being with consciousness. It may be worthwhile to show a way out of the difficulty which we encountered.

The simplest way out of the difficulty is to accept the conclusion which forced itself on us: to assume that the joint system of friend plus object cannot be described by a wave function after the interaction—the proper description of their state is a mixture.[13] The wave function is $(\psi_1 \times \chi_1)$ with a probability $|\alpha|^2$; it is $(\psi_2 \times \chi_2)$ with a probability $|\beta|^2$. It was pointed out already by Bohm[14] that, if the system is sufficiently complicated, it may be in practice impossible to ascertain a difference between certain mixtures, and some pure states (states which *can* be described by a wave function). In order to exhibit the difference, one would have to subject the system (friend plus object) to very complicated observations which cannot be carried out in practice. This is in contrast to the case in which the flash or the absence of a flash is registered by an atom, the state of which I can obtain precisely by much simpler observations. This way out of the difficulty amounts to the postulate that the equations of motion of quantum mechanics cease to be linear, in fact that they are grossly non-linear if conscious beings enter the picture.[15] We saw that the linearity condition led uniquely to the

[13] The concept of the mixture was put forward first by L. Landau, Z. *Physik*, 45, 430 (1927). A more elaborate discussion is found in J. von Neumann's book (footnote 4), Chapter IV. A more concise and elementary discussion of the concept of mixture and its characterisation by a statistical (density) matrix is given in L. Landau and E. Lifshitz, *Quantum Mechanics* (London: Pergamon Press, 1958), pp. 35-38.

[14] The circumstance that the mixture of the states $(\psi_1 \times \chi_1)$ and $(\psi_2 \times \chi_2)$, with weights $|\alpha|^2$ and $|\beta|^2$, respectively, cannot be distinguished in practice from the state $\alpha(\psi_1 \times \chi_1) + \beta(\psi_2 \times \chi_2)$, if the states χ are of great complexity, has been pointed out already in Section 22.11 of D. Bohm's *Quantum Theory* (New York: Prentice Hall, 1951). The reader will also be interested in Sections 8.27, 8.28 of this treatise.

[15] The non-linearity is of a different nature from that postulated by W. Heisenberg in his theory of elementary particles [cf., e.g., H. P. Dürr, W. Heisenberg, H. Mitter, S. Schlieder, K. Yamazaki, Z. *Naturforsch.*, 14, 441 (1954)]. In our case the equations giving the time variation of the state vector (wave function) are postulated to be non-linear.

unacceptable wave function $\alpha\,(\psi_1 \times \chi_1) + \beta\,(\psi_2 \times \chi_2)$ for the joint state. Actually, in the present case, the final state is uncertain even in the sense that it cannot be described by a wave function. The statistical element which, according to the orthodox theory, enters only if I make an observation enters equally if my friend does.

It remains remarkable that there is a continuous transition from the state $\alpha(\psi_1 \times \chi_1) + \beta(\psi_2 \times \chi_2)$ to the mixture of $\psi_1 \times \chi_1$ and $\psi_2 \times \chi_2$, with probabilities $|\alpha|^2$ and $|\beta|^2$, so that every member of the continuous transition has all the statistical properties demanded by the theory of measurements. Each member of the transition, except that which corresponds to orthodox quantum mechanics, is a mixture, and must be described by a statistical matrix. The statistical matrix of the system friend-plus-object is, after their having interacted ($|\alpha|^2 + |\beta|^2 = 1$),

$$\left\| \begin{matrix} |\alpha|^2 & \alpha\beta^* \cos\delta \\ \alpha^*\beta\cos\delta & |\beta|^2 \end{matrix} \right\|$$

in which the first row and column corresponds to the wave function $\psi_1 \times \chi_1$, the second to $\psi_2 \times \chi_2$. The $\delta = 0$ case corresponds to orthodox quantum mechanics; in this case the statistical matrix is singular and the state of friend-plus-object can be described by a wave function, namely, $\alpha(\psi_1 \times \chi_1) + \beta(\psi_2 \times \chi_2)$. For $\delta = \frac{1}{2}\pi$, we have the simple mixture of $\psi_1 \times \chi_1$ and $\psi_2 \times \chi_2$, with probabilities $|\alpha|^2$ and $|\beta|^2$, respectively. At intermediate δ, we also have mixtures of two states, with probabilities $\frac{1}{2} + (\frac{1}{4} - |\alpha\beta|^2 \sin\delta)^{\frac{1}{2}}$ and $\frac{1}{2} - (\frac{1}{4} - |\alpha\beta|^2 \sin^2\delta)^{\frac{1}{2}}$. The two states are $\alpha(\psi_1 \times \chi_1) + \beta(\psi_2 \times \chi_2)$ and $-\beta^*(\psi_1 \times \chi_1) + \alpha^*(\psi^2 \times \chi^2)$ for $\delta = 0$ and go over continuously into $\psi_1 \times \chi_1$ and $\psi_2 \times \chi_2$ as δ increases to $\frac{1}{2}\pi$.

The present writer is well aware of the fact that he is not the first one to discuss the questions which form the subject of this article and that the surmises of his predecessors were either found to be wrong or unprovable, hence, in the long run, uninteresting. He would not be greatly surprised if the present article shared the fate of those of his predecessors. He feels, however, that many of the earlier speculations on the subject, even if they could not be justified, have stimulated and helped our thinking and emotions and have contributed to re-emphasize the ultimate scientific interest in the question, which is, perhaps, the most fundamental question of all.