Quantum Models of Consciousness

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ABSTRACT

In this paper a short review of quantum models of consciousness is presented. In the conclusions two criteria are offered in order to classify these models, and a choice is operated towards those models which integrate quantum mechanics with special relativity.

1. CHRONOLOGICAL ORDER OF QUANTUM MODELS OF CONSCIOUSNESS

1.1. Alfred Lotka: Planck’s constant and the objective and subjective consciousness - 1925.

Lotka (1925) suggested his model before Heisenberg’s uncertainty principle and the formulation of the interpretation of Copenhagen. Lotka’s model can be considered the first quantum model of consciousness. Lotka suggested the existence of two types of consciousness. The first one, which he called deterministic, corresponds to the objectivity of the external world; the second one, which he called subjective, corresponds to the inner world (qualia). According to Lotka, consciousness of the first type (deterministic) is relative to all those phenomena which take place above Planck’s constant. At this level it was considered that the deterministic laws of classical physics took place. Consciousness of the second type (subjective) would, on the contrary, take place below Planck’s constant, where the deterministic laws of classical physics do not operate.
1.2. Niels Bohr model: consciousness creates reality through the collapse of the wave function - 1930.

The interpretation of Copenhagen proposed by Niels Bohr and Werner Heisenberg, suggests a direct link between consciousness and Quantum Mechanics. This interpretation attributes to consciousness an explicit role, through the act of measurement and observation, forcing the wave function to collapse into a particle and determining in this way reality itself. According to Bohr and Heisenberg consciousness would be an immanent property of reality which exists before the creation of reality.

1.3. Luigi Fantappiè: advanced waves and syntropy - 1941.

Fantappiè (1991) starts from the d’Alembert operator which in Klein-Gordon’s relativistic generalization of Schrödinger’s wave equation has two wave solutions: *retarded waves* which diverge from the past to the future and *anticipated waves* which diverge from the future to the past, and which for us, moving forward in time, correspond to converging waves, attractors/absorbers. Studying the mathematical properties of these waves Fantappiè discovered that the diverging waves are governed by the law of entropy, whereas the converging waves are governed by a symmetrical law, which concentrates energy, produces differentiation, structures and order, and which Fantappiè named syntropy. Fantappiè recognized the properties of the law of syntropy in the living systems, and concluded that living systems are a consequence of anticipated waves. Starting from the mathematical properties of the laws of syntropy and entropy, Fantappiè arrived to the formulation of a model of consciousness based on the following elements:

- **Free will**: which is generally considered a fundamental element of consciousness, is seen as the consequence of a permanent state of choice between information coming from the past and emotions coming from the future.
• Feeling of life: another basic component of consciousness is, according to Fantappiè, a direct consequence of waves moving backward in time, from the future to the past. Fantappiè says that when physical senses vanish, as in deep meditation, people experience states of consciousness in which past, present and future coexist. The coexistence of past, present and future would be a direct consequence of the coexistence of advanced and retarded waves.

• Non local memory: Fantappiè suggests the existence of non local correlations in the universe, as a consequence of the existence of advanced waves. For example, in living systems, memory could be a manifestation of non local links with past events which according to Klein-Gordon’s equation are still existing.

Fantappiè’s model is similar to the transactional interpretation of quantum mechanics and to the model of consciousness suggested by Chris King (1989).


According to Culberston (1963) memory, which is at the basis of consciousness, is a consequence of the change of state of matter in time. In other words, memory is equivalent to different states of matter in time. From this point of view memory is not formed by data stored in the brain, but by links between different states of space-time, and consciousness would therefore be located outside the brain, in the space-time. In order to describe this concept Culberston coined the term psycho-space. All objects of nature change their state in time. Therefore, according to Culberston, all objects of nature could be endowed with consciousness.

1.5. Horoomi Umezawa and Luigi Maria Ricciardi: Quantum Field Theory (QFT) – 1967

Ricciardi and Umezawa (1967) suggested a model of consciousness base on Quantum Field Theory (QFT). This model starts from the work carried by Fröhlich on Bose-Einstein condensates. The functions of the brain are seen as a consequence of collective quantum order states. For example, memory is
associated to “vacuum states”. In QFT vacuum states are the lowest level of energy in which, by definition, no particle is present. The stability of vacuum states make them ideal as a unit of memory. Umezawa and Ricciardi underline that one of the properties of vacuum states is that of developing correlations and order which can extend to the macroscopic level, producing fields which can effect the neural system. According to Umezawa, consciousness would be the result of the sum of quantum processes, while the neural systems would be limited to the transmission of macroscopic signals. The model of Umezawa was further developed by Giuseppe Vitiello.


Bose-Einstein condensates are a state of matter which is reached at extremely low temperatures. These condensates show highly ordered structures which behave as one particle. Fröhlich (1968) found that during digestion the dipoles of cells align and oscillate in a perfectly coordinated manner. Fröhlich suggested that this behaviour could lead to the formation of Bose-Einstein condensates at biological temperature. One important property of Bose-Einstein condensates is that they are able to amplify signals and code information; elements that according to Fröhlich are at the basis of consciousness. The work of Fröhlich inspired the model QBD (Quantum Brain Dynamics) of consciousness suggested by Umezawa and Ricciardi.


In quantum mechanics the tunneling effect is a phenomena in which a particle violates the principles of classical mechanics by passing through a potential barrier higher than the kinetic energy of the particle. According to Walker (1970), thanks to the tunneling effect electrons can pass from a neuron membrane creating virtual neural networks, parallel to the macroscopic network, and to which consciousness would be associated. The macroscopic neural system operates through synaptic messages, while the virtual neural system would operate through quantum synaptic tunneling effects. The real neural system would follow the laws of classical physics, while the
virtual neural system would follow the laws of quantum mechanics. Consciousness would be the product of the laws of quantum mechanics, even though the behaviour of the brain can be described using classical physics.


Pribram (1971) suggested a holographic hypothesis of memory and mind. An hologram is a 3D photography produced with the use of the laser. In order to produce an hologram, the object is first lighted with a laser light, then a second laser light creates interference pattern with the first one and the interference pattern is stored in the photographic film. When the film is developed only the interference pattern is revealed, but when this pattern is lighted with a laser light, the original 3D object appears. When the holographic film is split in half and then lighted with a laser light, each part continues to contain the original 3D image. Even when the film is divided in minuscule fragments, each fragment will contain the original 3D image. Differently from classical photography, each part of a holographic film contains all the information. According to Pribram, memory is not stored in a specific location of the brain, but works as a hologram. Only the interference among waves is stored. According to this view each information is transformed by the brain in a wave, and all the waves interfere giving place to holograms. According to Pribram, the same equations used to analyze holograms (Fourier transformations) are used by the brain to retrieve memory.


Bohm (1980) introduced the concepts of implicate and explicate order. In the implicate order there is no difference between mind and matter, while in the explicate order mind and matter are separated. When we deal with quantum mechanics the implicate order prevails, while when we deal with classical physics, the macroscopic laws, the explicate order prevails. According to Bohm, consciousness coincides with the implicate order. In the implicate order particles are in-formed, take form, through the collapse of the wave function, and the implicate order coincides therefore with the process of in-formation, taking
form. Bohm used the example of music in order to explain the implicate order. When we listen to a piece of music we perceive the implicate order, the information associated to the subjective experience of music, and not only the explicate order (physical parameters of sound waves, such as the frequency and amplitude). According to Bohm each material particle contains a rudimental form of consciousness. The process of information constitutes the bridge between mental and material properties of particles. At the lowest level of matter, the quantum mechanical level, the mental (conscious) and physical processes would coincide.


According to Herbert (1987) consciousness is a property which pervades all nature, a basic force of the universe. Herbert reaches this conclusion analyzing the probability principle, the existence of matter (objects are formed only when they are observed) and interconnection (entanglement). According to Herbert, these three principles are directly linked to the fundamental characteristics of consciousness: free will, ambiguity and interconnection.


John Carew Eccles, received the Nobel Prize in physiology in 1963 for discovering that chemical transmission of neural signals in vesicles presinaptic reticules determines only one exocytosis. Exocytosis is the unitary fundamental activity of the cerebral cortex and seems to obey a law of conservation. With quantum mechanics it is possible to explain this law of conservation introducing specific particles, which Eccles (1989) named psychons. According to Eccles psychons would be units of consciousness, which would connect together in order to produce a unitary experience and experiments show that exocytosis is governed by quantum processes. Eccles believes that mind is a non-material field, analogous to a probability field and that consciousness is triggered by the exocytosis process.

Marshall (1989) starts from Fröhlich’s work and links the holistic properties of consciousness to the behaviour of Bose-Einstein condensates. According to Marshall, conscious experiences arise when condensates are excited by electrical fields. Marshall believes that the collapse of the wave function always tends towards the formation of Bose-Einstein condensates and that a universal tendency towards the formation of life exists (anthropic principle). Mutations would not be casual, but would tend towards the formation of conscious life. The mental world (the conscious experience) would coincide with bosons (cohesive particles, such as gravitons and gluons which establish relations and can share the same states), while the material world would coincide with fermions (electrons, protons, neutrons) which do not share the same states.


Lockwood (1989) refers explicitly to the Many Worlds Interpretation of quantum mechanics. According to Lockwood, feelings are intrinsic attributes of physical states of the brain and would all coexist together, giving place to as many minds as all the possible combinations of these intrinsic attributes. Consciousness would then select the feelings and the mind, but would not create them.


Penrose (1989) starts from the assumption that reality is based on three worlds: the platonic world, physical and mental worlds. While in the Copenhagen interpretation the physical world is a consequence of the mental world (observation) through the collapse of the wave function, in Penrose’s model these worlds are separate and interact together through the collapse of the wave function. Hameroff and Penrose (1995) suggest that in microtubules, which are components of the cytoskeleton, every 25 msec the collapse of the wave function takes place, producing a conscious experience which puts the mental world in
relation with the physical world. According to these authors the sum of the wave collapses would generate the flux of consciousness and an orchestration of organized states (Orch-OR)


Chris King (1989) suggests a model based on the transactional interpretation of quantum mechanics (Cramer 1986) which starts from the dual solution of the relativistic wave equation (Klein-Gordon equation).

King states that quantum entities are always faced with bifurcations between past and future causes which require choices to be performed. King refers to the works of Eccles, Penrose and Hameroff works which show the relevance of quantum processes in living systems and suggests that according to the dual solution of the wave equation all living systems would be faced with bifurcations between causality and retrocausality. King names supercausality the sum of causality and retrocausality. These bifurcations would force living systems to operate choices which, according to King, is a common state to all living systems and to all the levels of organization of biological matter, from microstructures to macrostructures. As a consequence of the fact that all biological systems would act according to free will, and as a consequence of the fact that the outcome of free will is not determinable, biological systems should always show chaotic dynamics which cannot be studied in a deterministic or computational way.

King suggests two levels of explanation of consciousness. On the first level information moves from the mind to the brain as a consequence of the exercise of free will; on the second level, information flows from the brain to the mind thanks to the amplification of signals performed by the chaotic dynamics of the brain, which usual take the form of fractals.


Pitkänen (1990) TGD model of consciousness is based on the hypothesis of quantum jumps which involve different moments of time (quantum jumps between quantum histories) and on the concept that everything is consciousness. According to this model the universe starts from an initial state of maximum
consciousness which diminishes each time an interconnection (entanglement) is formed. In other words, consciousness would exist until it is not entangled, and consciousness can only be lost.

1.17. **Alex Kaivarainen: hierarchical model of consciousness - 1992.**

Kaivarainen (1992) starts from the assumption that neural excitation depends on 4 waves:

- thermal (de Broglie waves – B waves)
- electromagnetic (IR waves)
- acoustical (tr waves)
- gravitational (lb waves).

For this purpose 4 new types of particles are introduced:

1. **Effectons** (waves tr and lb);
2. **Convertons** (interaction of tr and lb);
3. **Transitons** (tr and lb states);
4. **Deformons** (superposition of transitions and convertons).

According to Kaivarainen this model could justify condensates at biological temperatures.


Stapp (1993) based his model on the assumption that consciousness creates reality (Copenhagen interpretation). Stapp developed Von Neumann’s idea that the objective universe is a consequence of the subjective one, and concluded that only the subjective experience is real, and that we can only know our subjective perceptions. Stapp model is based on three elements:

1. Reality is a consequence of finite events in the brain.
2. Each event produces an increase in consciousness.
3. Consciousness is the consequence of systems which observe.

Yasue and Jibu (1995) started their work from Umezawa’s model and developed a model in which brain waves are described using Schrödinger’s wave equation and the brain is treated as a macroscopic quantum system. Yasue suggests that consciousness arises from the interaction of electromagnetic fields and molecular fields of water in proteins. The evolution of the neural wave equation would not be random, but would tend to optimize the function of neurons. Yasue suggests a cybernetic model of consciousness which is based on the interaction among waves, and not on the physical neural network.


Vitiello (1995) developed a model based on QFT (Quantum Field Theory) which starts from Umezawa’s works, in which “vacuum states” were considered as unities of memory. Vitiello underlines that a problem with Umezawa’s model is that new information overwrites old information, losing old information. In order to overcome this limit Vitiello suggested a dissipative model of consciousness, in which living systems behave as dissipative systems in order to maintain inner temperatures compatible with the requirements of vacuum states (vacuum states require extremely low temperatures). When the brain is described as a dissipative system, it is necessary to introduce, in the mathematical formalism of QFT, the environment which absorbs the energy dissipated by the brain. According to this mathematical formalism, the environment, in order to be able to absorb energy, must have a backward in-time direction, opposite to that of the brain. The cognitive function of the brain that Vitiello tries to explain is memory, which by definition is a irreversible process which moves from the past to the future. As a consequence the environment must move backwards in time, from the future to the past. Vitiello’s model allows to increase the degrees of freedom and the size of information which can be stored in vacuum states, solving in this way the overwriting problem of Umezawa’s model. Vitiello concludes that consciousness derives from the constant interaction of the brain with its double, which is the environment.

Bondi (1998) studied those situations during which consciousness vanishes, for example sleep, anesthesia and pathological conditions. These conditions (physiological, pharmacological and pathological), would demonstrate the global nature of consciousness which turns on and off when the neural structure (synaptic junctions) lose their computational property of quantum origin. The model suggested by Bondi is based on the existence of channels which spiral through the labyrinth structures of the cortical system creating a flux of particles which determine a constant state of consciousness.


Hu and Wu (2002) underlines that spin is becoming the key element of quantum mechanics. Hu associates each spin to a mental pixel. The model of consciousness which is derived has non-local and non-computational properties.


Flanagan (2003) suggests that the ultimate level of description of mind and brain ought to be the quantum level, insofar as quantum field processes mediate all chemical and biological processes. Flanagan explores a number of possibilities:

- whether quantum fields constitute the ultimate level of description of biological systems;
- whether this ultimate level is nested within higher levels of organization by way of the kinds of self-similarity found in neural structures;
- whether this lowest level embodies the initial conditions upon which the brain exhibits sensitive dependence.

The thesis is that perceptual fields are coordinated with photon fields. Therefore the irreducibility of the secondary qualities flows from their elemental character and this might be in the form of:
• internal state spaces of gauge theory;
• additional spatial dimensions of string/M-theory;
• hidden variables theory.

Flanagan argues that colour vectors, considered as “immediately experienced qualities” together with their configurations in the visual field, ought to map to photonic vectors and their configurations in a photon field: vectors would get mapped to vectors, and fields to fields.


One way to avoid the Copenhagen/von Neumann metaphysics, while retaining a physical sense for the "collapse", is the concept of "decoherence". At first sight any isolated (relatively) small microscopic system would be "coherent", in the sense that entanglement of particles/waves would be the rule, and any (relatively) large macroscopic system would be "decoherent", in the sense that any property of this system, available to macroscopic observation and experimentation, would reveal no signal of entanglement between its elements. A widespread belief used to explain why nature behaves this way is the so-called "law of large numbers", that identifies the size of the system as the critical parameter for statistical cancellations to apply, thus generating a most probable behaviour that would correspond almost exactly to the predictions made by classical physics. In this view, the existence of quantum coherence at the macroscopic level would be extremely improbable, and could not frequently and continuously occur in the conscious brain.

For the defenders of the quantum mind hypothesis, or even for physicists who have worked with quantum macroscopic effects, there must be something wrong with such usage of the "law of large numbers". Therefore, if the "law of large numbers" is not universal and/or is not really a physical law, a theoretical possibility is open, for a large macroscopic system as the brain, to be continuously generating quantum coherent processes among particles/waves distributed along its volume. These processes would function as a "second-order brain", controlling and being controlled by classical processes that occur in that
same system (the "first-order brain"). This "second-order brain" would be the ultimate support for the flux of experiences called "the conscious mind".


Järvilehto (2004) argues that the understanding of the ultimate essence of matter depends essentially on the conception of human consciousness. In the framework of the organism-environment theory consciousness developing with human cooperation and communication is the basis of any description of the world. Järvilehto claims that the properties of matter are relations created in human action/measurement. Therefore, there is no ultimate essence of matter, and the question "What exists?" is identical with the question “What can we know?”

According to the organism-environment theory the world that may be described appeared with the appearance of human consciousness. Consciousness was created in a system of several individuals when their actions were joined in the achievement of common results. This joining was possible through communication which later developed to language. As communication was needed primarily for the production of common results, language developed primarily for the description of the common results that were intended or achieved. This means words were not, in the first hand, used for the description of the world, but they rather act as indicators of common results. Therefore, language (and also personal consciousness mediated by the language) was primarily directed towards generation of the cooperative organization. Thus, a word is basically not a symbol representing something, but a proposal for common action. It is precisely the common result that is stored in language, and, therefore, the language reflects the history of mankind and its culture. Language is the historical collection of the results of human cooperation. Those parts of the universe which became objects of language were primarily those that were needed by human beings in their actions in relation to their bodies. Therefore, the perceived (conscious) structure of the world reflects more the structure of the human body, as it may fit the environment, than any independent structure of the world. The implicate order of the universe for man is the implicate order of the human body in its environment.

Baaquie and Martin (2005) consider the human psyche to be part of a universal phenomena, and not specific and unique only to human beings.

Human psyche is described as a superposed state. In quantum mechanics, the spin of an electron has two forms of existence, namely the physical and the virtual. When it is observed it is in a physical state in which the spin points either up or down. On the other hand, if it is not being observed the spin is in a virtual state in which the spin can simultaneously exist in two mutually exclusive states. Every time the spin is observed, it is found to point in only one direction. Hence the virtual state can never be directly observed. Nevertheless only the human mind can see the virtual state. It follows that the human psyche itself must possess a similar virtual state.

In order to describe the human psyche two kinds of quantum fields are used:

1. one localized that refers to the specific individuality of the person;
2. the other which can overlap and include other's consciousness and requires a quantum superfield.

1.27. Donald Mender: Eccentrically Subjective Reduction (ESR) - 2007

Donald Mender (2007) underlines that physicists today have two tools for comprehending the universe of observed physical phenomena:

1. the standard model of quantum field theory (QFT)
2. general relativistic gravitation, which lies outside the quantum domain.

Recent developments, including superstring theory, loop quantum gravity, twistor geometry, show that the behaviour of gravitons, the vehicles of quantum gravity, has robust affinities with key properties of consciousness displayed only fragmentarily by either QFT or general relativity alone.
Roger Penrose and Stuart Hameroff postulate that, within the brain, physical "objects" constituted by gravitational field configurations mediate the self-induced collapse of linearly superposed, macroscopically diffuse quantum wavefunctions into noncomputationally selected component states.

A post-objective science of consciousness, may require decentring the very plurality of physical observations themselves, beyond even the statistical influence of standard QFT. Specifically, the structure of quantum gravitational self-operators may change to include individual, independent acts of measurement.

The Penrose-Hameroff hypothesis might benefit from such a multi–subjective, de-centred approach to measurement. The model would be enhanced and could be renamed Eccentrically Subjective Reduction (ESR).


Although experiments to detect faster-than-light particles have not been successful so far, recently, there has been renewed interest in tachyon theories in various branches of physics. Hari (2008) suggests that tachyon theories may be applicable to brain physics. Eccles proposed an association between psychons and dendrons which are basic anatomical units of the neocortex for reception. Hari proposes that a zero-energy tachyon could act as a trigger for exocytosis, not merely at a single presynaptic terminal but at all selected terminals in the interacting dendron by momentarily transferring momentum to vesicles, thereby decreasing the effective barrier potential and increasing the probability of exocytosis at all boutons at the same time. This is consistent with the view of tachyons, which treats them as strictly non-local phenomenon produced and absorbed instantaneously and non-locally by detectors acting in a coherent and cooperative way.

2. DISCUSSION

The models which have been described in this review can be divided in three main categories:
1. models which assume that consciousness creates reality and that consciousness is an immanent property of reality;
2. models which link consciousness to the probabilistic properties of quantum mechanics;
3. models which attribute consciousness to a principle of order of quantum mechanics.

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<th>1) Consciousness creates reality</th>
<th>2) Probability</th>
<th>3) Order principle</th>
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Table 1: classification of quantum models of consciousness

In table 1 each model is associated to one of these three categories. Analyzing the quantum models of consciousness which belong to the first category a tendency towards mysticism can be observed. All these models start from the Copenhagen interpretation of quantum mechanics and assume that consciousness itself determines reality. These models try to describe the creation of reality as a consequence of panpsychism, and assume that consciousness is an immanent property which precedes the formation of reality. The concept of panpsychism is explicitly used by most of the authors of this category. These assumptions cannot be falsified or tested in an experiment.
Analyzing the quantum models of consciousness which belong to the second category, also in this case, it arises that it is impossible to falsify the assumptions or test the models using experiments, as they consider consciousness to be linked to a realm, for example that of the Planck’s constant, which cannot be observed by modern science.

Analyzing the third group of models which attribute consciousness to principles of order which have been already discovered and used for physical applications (laser, superconductors, etc.) it is possible to imagine experimental tests which could falsify them. It is though important to note that many of these models require conditions which are not compatible with the characteristics of biological systems. The order principles on which most of these models are based require extreme physical conditions such as, for example, absolute zero temperatures (-273 C°).

Using together the criteria of scientific falsification and the criteria of biological compatibility only the models suggested by Luigi Fantappiè and Chris King survive this selection. It is interesting to note that these models are not pure quantum mechanical models, as they both originate from the generalization of Schrödinger’s wave equation (quantum mechanics) with special relativity.

3. CONCLUSION

In conclusion, it seems that all the models of consciousness which start from quantum mechanics cannot be translated into experiments, either because they cannot be falsified or because they are not compatible with the requirements of biology. The only two models which offer the possibility to be translated into experiments are those which unite quantum mechanics with special relativity.

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