14 Consciousness and Logic in a Quantum-Computing Universe

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Summary. The early inflationary universe can be described in terms of quantum information. More specifically, the inflationary universe can be viewed as a superposed state of quantum registers. Actually, during inflation, one can speak of a quantum superposition of universes. At the end of inflation, only one universe is selected, by a mechanism called self-reduction, which is consistent with Penrose's objective reduction (OR) model. The quantum gravity threshold of (OR) is reached at the end of inflation, and corresponds to a superposed state of 10^9 quantum registers. This is also the number of superposed tubulins – qubits in our brain, which undergo the Penrose–Hameroff orchestrated objective reduction, (Orch OR), leading to a conscious event. Then, an analogy naturally arises between the very early quantum-computing universe, and our mind. In fact, we argue that at the end of inflation, the universe underwent a cosmic conscious event, the so-called "Big Wow", which acted as an imprinting for the future minds to come, with future modes of computation, consciousness and logic. The postinflationary universe organized itself as a cellular automaton (CA) with two computational modes: quantum and classical, like the two conformations assumed by the cellular automaton of tubulins in our brain, as in Hameroff's model. In the quantum configuration, the universe quantum-evaluates recursive functions, which are the laws of physics in their most abstract form. To do so in a very efficient way, the universe uses, as subroutines, black holes – quantum computers and quantum minds, which operate in parallel. The outcomes of the overall quantum computation are the universals, the attributes of things in themselves. These universals are partially obtained also by the quantum minds, and are endowed with subjective meaning. The units of the subjective universals are qualia, which are strictly related to the (virtual) existence of Planckian black holes. Further, we consider two aspects of the quantum mind, which are not algorithmic in the usual sense: the self, and mathematical intuition. The self is due to a reversible self-measurement of a quantum state of superposed tubulins. Mathematical intuition is due to the paraconsistent logic of the internal observer in a quantum-computing universe.

14.1 Introduction

Consciousness... is the phenomenon
Whereby the universe's very existence is made known

Roger Penrose
The Emperor's New Mind

What is consciousness? Everybody knows about his/her own consciousness, but it is nearly impossible to communicate our subjective knowledge of it to others. Moreover, a complete scientific definition of consciousness is still missing. However, quite recently, it has been realized that the study of consciousness should not be restricted to the fields of cognitive science, philosophy and biology, but enlarged to physics, more precisely, to quantum physics.

The most popular (and conventional) description of consciousness is based on the classical computing activities in the brain's neural networks, correlated with mental states. In this case, mind and brain are the same, and are compared to a classical computer. This approach (see, for example, [10, 11]) is called in various ways: physicalism, reductionism, materialism, functionalism, computionalism.

However, although the brain can actually support classical computation, there is an element of consciousness that is noncomputable (in the classical sense), as was shown by Penrose [36, 37]. Moreover, the seminal paper by Stapp [43, 44] clarified why classical mechanics cannot accommodate consciousness, but quantum mechanics can.

Finally, reductionism cannot explain the "hard problem" of consciousness, which deals with our "inner life", as illustrated by Chalmers [8].

A quite different line of thought about consciousness is the one that comprises panpsychism, pan-experientialism, idealism, and fundamentalism. Pan-experientialism states that consciousness (or better protoconsciousness) is intrinsically unfolded in the universe, and that our mind can grasp those proto-conscious experiences. This line of thought goes back to Democritus, Spinoza [42], Leibniz [29], until Whitehead [49] who reinterpreted Leibniz's "monads" as "occasions of experience". Shimony [41] compared Whitehead's occasions of experience to quantum jumps.

More recently, Penrose interpreted the occasions of experience as the quantum-state reductions occurring at the Planck scale, where spin networks [38, 39] encode protoconsciousness. This is a pan-experiential approach to consciousness that is consistent with quantum gravity, and is called "objective reduction" (OR) [36, 37]. A further development is the Penrose–Hameroff "orchestrated objective reduction" (Orch OR) [22, 23, 24], which deals with the self-collapse of superposed states of the tubulins in the brain. Superposed tubulin states are qubits, and perform quantum computation, until they reach the quantum gravity threshold, then they collapse to classical bits, giving rise to a conscious event.

Finally, Chalmers [8] claimed that physical systems that share the same organization will lead to the same kind of conscious experience (principle of organizational invariance). As physical systems that have the same organization (no matter what they are made of) encompass the same information, it follows, from the above principle, that information is the source of consciousness. The present chapter is consistent with Chalmers' conclusions. This is of course valid also for an immaterial system, like the vacuum-dominated early inflationary universe that, as was shown in [53], is a superposed quantum state of qubits.

At this point, a conjecture arises very naturally: the early universe had a cosmic conscious experience at the end of inflation [54], when the superposed quantum state of $n = 10^9$ quantum gravity registers underwent objective reduction. The striking point is that this value of n equals the number of superposed tubulins – qubits in our brain, which undergo orchestrated objective reduction, leading to a conscious event. Then, we conjecture that the early universe and our mind share the same organization, encompass the same quantum information, and undergo similar conscious experiences. In other words, consciousness might have a cosmic origin, with roots in the preconsciousness ingrained directly from the Planck time. In this context, we revisit the concept of qualia, and we interpret them as units of the universals, which are global properties of quantum-evaluated recursive functions at the Planck scale (the laws of physics in their most primordial form). In this way, the universals and qualia in particular, provide a bridge between our minds and the physical world.

The mathematical world is also committed with our consciousness: it deals with the most profound part of it, the self. Self-awareness and mathematical intuition are the outcomes of nonalgorithmic processes, which are due to self-measurements without decoherence, and to the sequent calculus of a paraconsistent, symmetric logic.

14.2 The "Big Wow"

Had I been present at the creation,
I would have given some useful hints
for the better ordering of the universe.

Alphonso the Wise
King of Castile & Leon
(1221–1284)

The hot Big Bang theory of the formation of the universe raises some problems that are (partially) solved by the inflationary theory [21]. The early inflationary universe is proposed to have had an accelerated expansion, the duration of which depends on the particular inflationary model. In any case, inflation is believed to have lasted an extremely short time: in the range of $10^{-33} - 10^{-35}$ s. In particular, in the chaotic inflationary model [32], inflation starts at the Planck time, $t_{\mathrm{P}} \approx 10^{-44}$ s. (In this chapter, we also consider inflation starting at the Planck time, and the end of inflation occuring at time $t \sim 10^{-34}$ s.)

During inflation, the universe behaves like a de Sitter space-time, which is empty (no matter, no radiation – it is a vacuum-dominated universe), expands exponentially, and has an event horizon.

In previous work, [52] we considered quantum de Sitter horizons and, by means of the holographic principle [26, 27, 46] and spin networks [38, 39], the early inflationary universe was described in terms of quantum information. How does it work? In this model, time is discrete, and is quantized in Planck time units: $t_n = (n+1)t_{\mathrm{P}}$ with $n = 0, 1, 2, 3\ldots$. At each time step, there is a de Sitter horizon with the quantized area of $A_n = Nl_{\mathrm{P}}^2$ (where $N = (n+1)^2$ and $l_{\mathrm{p}} \approx 10^{-33}$ cm is the Planck length). The holographic principle states that all the information enclosed in a region of space with volume V, is encoded on the surface S bounding V. More precisely, each pixel of area (a pixel is one unit of the Planck area) of S encodes one bit of information. In a more sophisticated (quantum) version of the holographic principle [53] the encoded information is quantum, and each pixel encodes one quantum bit (or "qubit") of information, which is a quantum superposition of the two classical bits, 0 and 1. By the holographic principle, then, it turns out that each de Sitter horizon's quantized area A_n encodes N qubits. At this point, every quantized horizon can be viewed as a quantum register (a quantum register is the memory of a quantum computer, built with qubits).

Through considerations of the actual entropy of our universe, (and of the maximal possible one), we found [54] that the amount of quantum information processed during inflation was $N \sim 10^{18}$ qubits, corresponding to the "selection" of the n-th $\sim 10^9$ quantum register. But what stopped the N-qubit quantum state from "decohering" if there was no environment surrounding it (the universe was empty)?

The n quantum registers $Q_1, Q_2, Q_3, \ldots Q_n$ built, respectively, with 1, 4, 9. . . ,10^{18} qubits can be thought to be in a superposed state like the "many-universes" interpretation of quantum mechanics [17]. In [54], a single "universe" is selected, the n-th $\sim 10^9$ quantum register, by a mechanism (self-decoherence), which is very similar to the objective reduction model (OR) of Penrose [36, 37] related to instability in the superposition/separation of the fundamental structure of the universe at the Planck scale.

In our case, the quantum gravity threshold of OR is reached at the end of inflation, with gravitational self-energy $E \sim 10^{10}$ GeV and preconscious time $T \sim 10^{-34}$, (corresponding to a superposed state of $n \sim 10^9$ quantum registers for a total of $N \sim 10^{18}$ qubits). According to the Penrose–Hameroff consciousness model of "orchestrated objective reduction" (Orch OR) [22, 23, 24], as calculated by the uncertainty principle, $\sim 10^9$ is the number of superposed tubulins proteins qubits in our brain that undergo Orch OR for conscious events of cognitive relevance, i. e. several hundred milliseconds, and that $\sim 10^{18}$ is the total number of tubulins protein qubits in our brain. Then, we suggested that at the end of inflation, the universe had a cosmic conscious experience (the "Big Wow", so renamed by Hameroff) and, according to Chalmers' principle of organizational invariance [8] an analogy naturally arises between the very early quantum-computing universe, and our conscious minds.

14.3 How the "Big Wow" Drove Human Minds

There is a coherent plan to the universe
Though I don't know what's a plan for

Fred Hoyle

Quantum gravity registers do perform quantum computation, but in a rather particular way, showing up some features of self-organizing systems. We recall that self-organization is a process of evolution taking place basically inside the system, with minimal or even null effect due to the environment.

In fact, the dynamical behavior of quantum gravity registers follows some cybernetic principles:

i) Autocatalytic growth

At each computational time step, the presence of a Planckian black hole (which acts as a creation operator), makes the quantum gravity register grow autocatalytically. As N qubits represent here a de Sitter horizon with an area of N pixels, the autocatalytic growth, in this case, is exponential expansion, i. e. inflation.

ii) Autopoiesis (or self-production)

The quantum gravity register produces itself. The components of the quantum gravity register generate recursively the same network of processes that produced them.

In this case recursion is defining the program in such a way that it may call itself. This is along the same line of thought as Kauffmann's "Fourth Law" [28]: "...The hypothesis that the universe as a whole might be a self-constructing coevolving community of autonomous agents that maximizes the sustainable growth...".

For Kauffmann, the autonomous agents are knotted structures created of spin networks that act on one another and become collectively autocatalytic. Our picture and Kauffmann's picture, are equivalent, because spin networks pierce the de Sitter horizons' surfaces [53, 52].

iii) Self-similarity

This model of the early inflationary universe is based on the holographic principle [26, 27, 46], in particular, on the quantum holographic principle [53]. But each part of a hologram carries information about the whole hologram. So, there is a physical correspondence between the parts and the whole.

iv) Self-reproduction

Can a quantum gravity register, as a unit, produce another unit with a similar organization? This possibility, which could be taken into account because the quantum gravity register is an autopoietic system, (and only autopoietic systems can self-reproduce), is in fact forbidden by the no-cloning theorem [50] (quantum information cannot be copied).

However, there is a way out. When the selected quantum gravity register collapses to classical bits, it is not just an ordinary quantum register that collapses, but an autopoietic one. The outcomes (classical bits) carry along the autopoiesis. The resulting classical automaton is then autopoietic and, in principle, can self-reproduce.

Moreover, Chalmers' principle of organizational invariance would assign to the (produced) unit with similar organization, the same amount of information, and the same conscious experience as the original one.

From the cybernetic principles, the organizational invariance principle and from the no-cloning theorem, we get *the principle of alternating computational modes*: "A unit produced by an autopoietic classical computing system built up from the outcomes of a decohered quantum autopoietic system, shares the same organization, the same amount of information, and the same conscious experience as the producing unit. Moreover, in order to share the same conscious experience as the decohered quantum system, the produced unit must alternate quantum and classical computational modes at least once".

The above arguments are summarized in the following scheme:

Autopoietic quantum register
→ **no-cloning theorem**
→ **no self-reproduction**
→ **decoherence**
→ **autopoietic classical cellular automaton**
→ **self-reproduction**
→ **produced unit with the same organization**
→ **principle of organizational invariance**
→ **the produced unit shares the same information content, and the same conscious experience**

→ **the produced unit gets both quantum and classical computational modes, the former from the autopoietic quantum register, the latter from the autopoietic classical cellular automaton**
→ **the modes alternate with respect to each other.**

Then, we are led to make the conjecture that the final outcome of a quantum gravity register might be a brain. In fact, tubulins in the brain alternate classical and quantum computational modes [22, 23, 24].

A related paper on the issue of a cybernetic approach to consciousness can be found in [19].

14.3.1 Entanglement with the Environment

This superposed state will collapse to classical bits by getting entangled with the emergent environment (radiation-dominated universe). This entanglement process with the environment can be interpreted as the action of an XOR (or controlled NOT) gate, as was illustrated in [53], which gives the output of the quantum computation in terms of classical bits: the source of classical information in the postinflationary universe.

14.3.2 Holography and Cellular Automata

Cellular automata (CA) were originally conceived by von Neumann [47], to provide a mathematical framework for the study of complex systems. A cellular automaton is a regular spatial lattice where cells can have any of a finite number of states. The state of a cell at time t_n depends only on its own state and on the states of its nearby neighbours at time t_{n-1} (with $n \in Z$). All the cells are identically programmed. The program is the set of rules defining how the state of a cell changes with respect of its current state, and that of its neighbors.

It holds that the classical picture of holography (given in terms of classical bits) can be described by a classical CA. The rules do force patterns to emerge (self-organization). By taking into account the “classical” holographic principle, we are led to believe that at the end of inflation, the universe starts to behave as a CA that self-organizes and evolves complexity and structure.

It should be noted that this CA is made out of the bits that are the outcomes of the collapse of the qubits of the quantum gravity register that is an autopoietic quantum system. Then, this CA is an autopoietic classical system.

There are two important consequences that follow.

i) The CA, being autopoietic, undergoes autocatalytic growth, and the classical universe is still expanding. However, as classical computation is slower than quantum computation, the expansion is no longer exponential (postinflationary universe).

ii) The CA, being a classical autopoietic system, can self-reproduce. According to the "principle of alternating computational modes" discussed in Sect. 14.3, the produced units will be able to perform both quantum and classical computation. We conclude by saying that in our model, the postinflationary, classically holographic universe, follows the laws of classical complex adaptive systems (systems at the edge of chaos).

14.4 Consciousness and Tubulins/Qubits

The state of least excitation
Of consciousness is the field
Of all possibilities

Maharishi Mahesh Yogi

So far, consciousness was studied in the context of neuroscience, and was described as an emergent feature of classical computing in the brain's neural networks. But neuroscience fails to explain some features of consciousness as, for example, subjective experience (Chalmers' "hard problem" [8]). A new, different approach to the study of consciousness is due to Hameroff and Penrose [22, 23, 24] and it is based on quantum effects occurring in tubulins. In a brain's neuron there is the cytoskeleton, which is made of protein networks. The most important components of the cytoskeleton are microtubules. Microtubules are hollow cylindrical polymers of proteins called tubulins. Tubulins have electrical dipoles and they can be in (at least) two different states (or conformations). Tubulins have been studied in classical computing. In fact simulations suggest that tubulins behave as a classical CA. But tubulins can also be in a superposition of two (or more) conformation states. In this case they represent qubits, and they behave as a biological quantum cellular automaton. Indeed, tubulins can perform both classical and quantum computing. In a classical computing mode, patterns of tubulins move, evolve, interact and lead to new patterns. Quantum coherence emerges from resonance in classical patterns. When the quantum gravity threshold is reached, self-collapse occurs and then tubulins evolve as a classical CA. In the orchestrated objective reduction (Orch OR) model of Penrose and Hameroff [22, 23, 24], the number of tubulins/cell involved in the threshold is $n = 10^9$, with a coherence time $T = 500\,\text{ms}$. As tubulins are qubits, we can indulge in speculating about the brain-universe, with $n = 10^9$ quantum gravity registers, and a coherence time $T = 10^{-34}$, which might have a conscious experience. Since the inflationary universe performed quantum computation, and was able to achieve consciousness, we might ask if this will be the case with any quantum computer? For the moment, the only possible answer is no, for three reasons:

1. Because quantum computers are very difficult to build in practice, as the technology is not yet so advanced to maintain coherence for a sufficiently long time.

2. Because quantum computers (at least the first generations) will not have enough mass.
3. For a quantum system to be able to get a conscious experience, it is a necessary but not sufficient condition that it performs quantum-computation. The extra requirement is that the quantum-computing system must be quantum-autopoietic. However, we cannot really foresee anything definitive yet: In the long run quantum computers might have conscious experiences.

14.5 Consciousness Arises in the "Bits Era"

In our model, during the "qubit era" there are no events in the usual sense, (occasions of experience, in the philosophical language of Whitehead [49]). So, if we, Boolean-minded beings, conceive consciousness in terms of occasions of experience (events in the Boolean sense), we can argue that in the qubits era there was no consciousness at all in the universe (perhaps, there was just preconsciousness).

Consciousness appeared in the classical "bits era": it was the projection in the past of future internal observers who had to be programmed by the self-organizing CA, in order to observe the emergent events.

14.5.1 The Boolean Observer

The mind is like a parachute
It works only when it is open

Unknown

To observe the events in the postinflationary universe, the observers should be Boolean. This means that the qubits – tubulins of the observers' brain should collapse to classical bits at a rather high frequency. Of course, being the Boolean observers, they will not be able to grasp the unfolding quantum computing structure of space-time at the fundamental level (the Planck scale).

The problem is that a Boolean observer is endowed with the concept of time, which is a mere artifact of his/her own perception, and moreover, he/she tends to extend this concept to regions of reality where it is meaningless. A Boolean observer, is a classical logician, like Aristotle. He/she is not capable of putting himself/herself in relation with a quantum system without the mediacy of an external quantum observer. Even worse, he/she will never be able to recognize the quantum universe as a whole, as an internal observer [3]. This would lead him/her into a contradiction with himself/herself. Eventually, he/she will be replaced by a double logic-minded observer in Sect. 14.6.

14.5.2 The Analogy

Inflation (the qubit era) is for the universe what preconsciousness (superposed tubulins) is for our mind/brain. The end of inflation (beginning of the "bits era") is for the universe what consciousness (Orch OR of superposed states of tubulins) is for our mind.
The analogy goes like this:

For tubulins in the brain:
CLASSICAL CA
→ EMERGENCE OF QUANTUM COHERENCE (PRECONSCIOUSNESS)
→ QUANTUM CA
→ SELF-COLLAPSE BY ORCHESTRATED OBJECTIVE REDUCTION
→ CONSCIOUS EXPERIENCE
→ CLASSICAL CA.

For qubits in the early universe:
CLASSICAL BIT (THE VACUUM)
→ HADAMARD QUANTUM LOGIC GATE
→ QUBIT
→ BEGINNING OF INFLATION (THE UNIVERSE IS A SUPERPOSED STATE OF QUANTUM REGISTERS)
→ SELF-REDUCTION BY OBJECTIVE REDUCTION (END OF INFLATION)
→ COLLAPSE OF QUBITS TO BITS (THE XOR GATE)
→ CONSCIOUS EXPERIENCE
→ CLASSICAL CA.

Of course, the analogy between our mind and the universe is very speculative at this stage, but the emergent picture is quite exciting: it seems that our mind/brain owes its structure and organization to the very early universe. This is in agreement with the Penrose–Hameroff belief that consciousness is a fundamental property of reality, and has its roots in the space-time structure at the Planck scale. Then, although we can be just classical as observers, we can be also quantum as thinkers. In fact, for example, we can conceive quantum computation. This fact must be the result of a kind of *imprinting* we received from the quantum-computing early universe. If we did not have both quantum and classical computational modes available in our brain, in other words, if we were always conscious and Boolean, we would not be able to think in a quantum mode.

14.6 The Double Logic of the Observer Inside a Quantum Universe

Logic is just the premise
Of wisdom,
Not the conclusion

Mr. Spock
Star Trek Enterprise

In a recent paper, [3] we inquired about the internal logic of a quantum computer, in the simplest case of one qubit. Standard quantum logic [4] in fact fails when it tries to describe a closed quantum system, like a quantum computer during the computational process. Then, standard quantum logic is confined to describe the projective measurement performed by an external observer, not the whole quantum computation, which appears as a black box to the external observer.

The alternatives to standard quantum logic are, for example paraconsistent logic, [12] linear logic [18] and basic logic [40]. In our approach, we illustrated, in logical terms, a reversible quantum measurement, with no hidden quantum information, performed by a hypothetical "insider observer" [56]. In our case, the reversible measurement was a purely theoretical tool to investigate the internal computational state. And the "insider observer" was a fictitious being inside the quantum computer, used to illustrate the quantum measurement scheme in a quantum-space background. The resulting logic of the insider observer is paraconsistent and symmetric, like basic logic.

In a paraconsistent logic, the well-known laws of noncontradiction and excluded middle do not hold. In our case, then, we admit a superposition of the opposite truth values 0 and 1, that is, true and false.

It was quite natural to wonder what would happen if the whole universe were a quantum computer, and the previously fictitious insider observer were instead a true human being. In this case, the observer would be internal with respect to the universe as a whole, and thus endowed with a paraconsistent logic, and external with respect to any other quantum subsystem, and thus endowed with standard quantum logic. We argue that in the former case, the mental states of the observer are superposed, while in the latter they are not. The corresponding "hardware" would be a quantum configuration of superposed tubulins, and a classical configuration, respectively, as in Penrose–Hameroff model.

The double-minded observer, apart from accommodating the Orch OR model of consciousness, has some other intriguing features, like a modification [55] of Goedel's first incompleteness theorem [20].

14.7 IT from Qubit: The Whole Universe as a Quantum Computer

I've made such a terrible
Mess of things... and all I wanted
To do was to rule the universe

Unknown

We believe that the offspring of the universal cellular automaton (CA) discussed in Sect. 14.3.2, are quantum minds in the sense of Hameroff–Penrose [22, 23, 24], and black holes – quantum computers on a noncommutative geometry background [57]. The event horizon of such quantum black holes is the surface of a fuzzy sphere [33]. If the black hole – quantum computer is processing N qubits, its event horizon is a fuzzy sphere with $n = 2^N$ elementary cells. Each cell encodes a string of N bits. For example, a black hole – quantum computer with two qubits has an event horizon that is a fuzzy sphere with four elementary cells, each cell encoding one of the four states: $|00\rangle, |01\rangle, |10\rangle, |11\rangle$.

At first glance, one might visualize this as a classical CA, each cell encoding a string of bits. It should be noticed, however, that due to the noncommutative structure of the background geometry, these basis states can be superposed and entangled! At the end of computation, the black hole emits one string of N bits (for example $|00\rangle$, for $N = 2$) and the corresponding cell evaporates. The end of a quantum computation corresponds to decoherence of the quantum computer. If we suppose that these quantum black holes undergo OR, we are surprised to find very long decoherence times, as we show in what follows.

The area of a fuzzy quantum black hole is given by [30]: $A = NA_{\mathrm{P}}$ where N is the number of qubits and A_{P} is the Planck area: $A_{\mathrm{P}} = l_{\mathrm{P}}^2$, where $l_{\mathrm{P}} \approx 10^{-33}$ cm is the Planck length. For a black hole, the following relation holds between its mass and the surface area of its event horizon: $M = \sqrt{A}$. In the case of fuzzy black holes, one has then: $M = \sqrt{N} l_{\mathrm{P}}$. The OR decoherence time T is given by: $T = \frac{\hbar}{E}$, where E is the mass energy $E = mc^2$. In our case we have: $T = \frac{1}{\sqrt{N} t_{\mathrm{P}}}$ (with $\hbar = c = 1$), where $t_{\mathrm{P}} \approx 10^{-43}$ is the Planck time.

One can easily calculate that for a black-hole encoding, for example, the same number of qubits as the average number of superposed tubulins in our brain, that is, 10^9, the decoherence time is about $T = 10^{31}$ s, which is a very long time, even compared to the age of our universe, that is $H^{-1} \approx 10^{17}$ s, where H is the Hubble constant. Of course, the bigger the number of qubits encoded by a black hole, the shorter will be its decoherence time.

Let us suppose that the whole universe alternates between two configurations, the first like a classical CA, the second like a fuzzy black hole – quantum computer, performing classical and quantum computational tasks, respectively. This behavior is similar to that of the CA of tubulins in our brain, that can undergo both quantum and classical configurations. In the quantum configuration, the universe decoheres by OR, and emits one classical string as its output, with the evaporation of the associated cell. However, the cell is instantaneously replaced, because of the universe's expansion. The age of the universe is, as we said above, 10^{17} s. Since the Big Bang, the universe encoded, in its cosmological horizon, 10^{120} qubits [58]. The area of the cosmological horizon is then: $A = 10^{120} A_{\mathrm{P}}$. If the whole universe can be described at present as a giant fuzzy black hole –quantum computer, its decoherence time is: $T_{\mathrm{U}} = \frac{1}{10^{60} t_{\mathrm{P}}} \approx 10^{-26}$ s.

As we see, the quantum universe has an extremely short decoherence time, and suddenly recomposes as a classical CA, because of its own expansion, and then reorganizes as a fuzzy quantum computer, and so on. One might argue that, due to the fact that the decoherence time of the universe is much smaller than its "dynamical" time: $T_{\mathrm{U}} \ll H^{-1}$, the universe should not be considered a quantum system. However, its quantum-computational efficiency is enormous, as in that very short lapse of time, the universe can perform a huge computational task at the Planck scale. In fact, at the Planck scale, space-time can quantum evaluate a composite function of depth $d = 10^{120}$ [58].

Moreover, it should be noticed that the decoherence time of the quantum state of superposed quantum universes at the end of inflation, was not calculated in this way, as that quantum system was not a fuzzy black hole – quantum computer.

14.8 Quantum Minds and Black – Hole Quantum Computers in a Quantum Game

Calculemus

G.W. Leibniz

Fuzzy quantum black holes and quantum human minds can be viewed as subroutines of the whole quantum-computing universe discussed in Sect. 14.7.

This cosmic network exploits quantum communication complexity, and, thanks to entanglement, allows its parts to accomplish a distributed task without the need of any kind of communication. This is possible because all its parts share a common quantum space-time background, which is nonlocal as it is pointless (in a fuzzy sphere, points are replaced by elementary cells). In this picture, the fuzzy black holes –quantum computers and the quantum minds who share prior entanglement, are the players of a quantum game, and do not necessitate to communicate among them (*pseudotelepathy* [6]).

The literary analog of this game played by minds and black holes might be the cosmic game (as far as minds are concerned, at least) considered in "The Glass Bead Game" [25], by Hermann Hesse. An interesting scientific/philosophical interpretation of that novel can be found in Zimmermann's paper [51]. In our case, all the players are quantum-evaluating recursive functions that are the laws of physics in their discrete and most primordial form [58]. More precisely, some elements of the net are quantum memory registers, others are scratch registers needed to store intermediate results. However, according to the history of the patterns of the original CA, the role of a register may change with time.

The global properties of the quantum-evaluated functions can be interpreted as the universals (universalia). More specifically, we will call the results obtained by black holes universalia in re (universals in the thing), the universal features of singular things, inherent in the things themselves. A human mind has a much shorter decoherence time than that of a fuzzy black hole –quantum computer. Then the results obtained by a human mind will be partial, with respect to black holes' results.

Then, after objective reduction, the quantum mind can start again the computational process in parallel with black holes, but the resulting knowledge of the physical laws will be partial, discontinuous, and random. Of course, the fact that the knowledge of the physical world can be acquired by us only partially, might disturb physicists who believe in a TOE (theory of everything). However, the process discussed above, allows humans to have at least a summary understanding of the physical world, which would be impossible otherwise. As Einstein noted: "The most incomprehensible thing about the world is that it is comprehensible".

The quantum mind outputs are to be considered as universalia post rem (universals after the thing), the concepts of the human mind regarded as posterior to the things represented by these concepts. So, universalia post rem are algorithmic in nature, and have a counterpart in the physical world. They belong to the set of computational aspects of consciousness. The bridge between our consciousness and the physical world is given by the common language of our minds and the universe. In fact, the outputs of all quantum-evaluated functions form a set of finite strings of bits that is a language based on the alphabet $\{0, 1\}$. Of course, this bridge is possible because our consciousness is, in some of its aspects, algorithmic.

As we will see in what follows, qualia are, in a sense, units of universalia post rem. There are, however, two aspects of consciousness, namely, mathematical intuition, and the self, which are not algorithmic.

14.9 Qualia and Quantum Space-Time

You have to ask children and birds
How cherries and strawberries taste

Goethe

What is the relation between the "occasions of experience" of Whitehead and the subjective aspects of conscious experience known as "qualia" (subjective conscious experiences), a term coined by Lewis [31]? In fact, what is the wider, basic field of protoconsciousness Whitehead talked about? The old question is: is there any explanatory bridge between brain functions and qualia? As was pointed out by Levine [30], there is an "explanatory gap": and it does not matter how well we can know the brain functions, this knowledge will not explain how a conscious experience is generated.

Chalmers [8] states that reductionism cannot explain the "hard problem" of consciousness, which deals with our "inner life": how can a conscious experience (a quale) emerge from a physical function of the brain? His way out to the explanatory gap is to consider the conscious experience as a fundamental entity. A theory of consciousness can then be built on fundamental entities, as in physics.

There are other authors who simply deny the existence of qualia, like Dennet [13], or say that the hard problem will never be solved, like McGinn [34], who claims that our mind is too limited to afford the problem. (Or: Why we cannot observe our inner-selves? Because there is not an observer inside the system: we are the system itself.) Why others cannot observe (test) our conscious experience from outside as well? Why the knowledge of consciousness cannot be objective? If we make the analogy between our inner life and the universe, then we get the answer, as we know that there is no definition of an external observer outside the universe. This does not mean, however, that the problem of consciousness is not a scientific one. It is a bit like the problems of quantum gravity (no possibility to test it directly) and quantum cosmology (no external observer). Quantum gravity is the theory that should reconcile quantum mechanics and general relativity. Even if one day we would be able to build the theory of quantum gravity, it will be a very peculiar theory: based mostly on mathematical consistency, but not directly testable, as it deals with space-time at the Planck scale.

It should be noted that the Planck scale couldn't be tested not only because at present we don't have at our disposal such a huge energy like the Planck energy, but also for impossibility a priori. In fact, at the Planck scale, space-time starts to lose its well-known smooth structure, and becomes a *marasmus* of virtual black holes and wormholes: the "quantum foam" [48]. Any attempt at probing space-time at this scale would then lead to outcomes belonging to another universe [59]. General relativity and quantum mechanics can be considered to be the two distinct offspring of quantum gravity at scales far above the Planck scale. They can both be tested directly (and separately), but their origin, quantum gravity, cannot. Following McGinn and Chalmers together, we would say then that consciousness exists, is fundamental, but its origin cannot be probed, just like Planck-scale physics. The quale and the Planck scale event might then be identified with the Kantian noumenon: the thing in itself. This should not be too surprising, in fact, protoconsciousness is rooted at the Planck scale which is not testable itself. But, why is protoconsciousness rooted at the Planck scale? Here we give an explanation slightly different from that of Penrose, although the two interpretations are closely related.

In quantum gravity, more precisely in loop quantum gravity, it is believed that quantum space-time is made of spin networks. This is the mathematical structure introduced by Penrose and developed by Smolin and Rovelli [38, 39] in the context of loop quantum gravity. However, it is also determined that quantum space-time is a quantum foam of virtual black holes and wormholes: this is Wheeler' s description [48] of the quantum foam. Can the two views go along together? We believe so, because of the following arguments.

Consider a macroscopic black hole. The edges of spin networks pierce the black-hole horizon and excite curvature degrees of freedom on the surface [1]. These excitations (microstates) account for the black-hole entropy that turns out to be a quarter of the area of the horizon, (in units of Planck area), in accordance with the holographic principle [26, 27, 46]. Moreover, the states that dominate the counting correspond to punctures of spin $j = 1/2$ and one can in fact visualize each microstate as a bit of information. The obvious generalization of this result is to consider open spin networks with edges labeled by the spin $-1/2$ representation of SU(2) in a superposed state of spin "up" and spin "down". The microstate corresponding to such a puncture will be a unit of area that is "on" and "off" at the same time, and it will encode a qubit of information [53].

Now, let us go back to the virtual black-holes in the quantum foam. Spin networks' edges pierce each virtual black-hole horizon in one point (puncture). The surface area of a virtual (Planckian) black hole is one pixel, (one unit of Planck area) and by the (quantum) holographic principle, it encodes one qubit of information. So, while in Penrose's view qualia can be identified with spin networks, in our view, qualia are the result of the action of spin networks on the quantum foam, that is, quantum information (qubits). We really think it is just a matter of interpretation, saying that qualia are spin networks or qubits encoded in virtual black holes' horizons pierced by spin network's edges.

McGinn [34] suggests that consciousness was present before the Big Bang, because the Big Bang is the beginning of space, and consciousness is nonspatial in nature. Although we also think that consciousness was indeed present in the early universe (as protoconsciousness), we wish to clarify that the problem of the nonspatial nature of consciousness is ill-posed. Consciousness itself distorts our understanding of space-time. Humans perceive space-time as a four-dimensional continuum, a smooth manifold. But space-time has a discrete structure, which becomes apparent at the Planck scale. At that scale, the familiar notion of an event like a point in a four-dimensional manifold loses its meaning. "Points" at the Planck scale are extended objects [57]. So, it is protoconsciousness that holds the right place in (quantum) space-time. Consciousness, the classical one we human beings deal with, appears to be nonspatial because (classical) space-time, as we understand it, is not the real thing. Qualia correspond to the computational outputs of fuzzy Planck-scale black hole –quantum computers, encoding one qubit of information. Planck-scale black holes are virtual objects, they are the constituents of the quantum foam [48]. Notice, however, that although being a virtual object, a Planckian black hole can be considered "eternal" as a quantum superposition: (its decoherence time is 10^{34} s, equal to the squared age of the universe). In a sense, a fuzzy Planckian black hole is the very quantum object. Qualia can be considered to be the "units" of universalia post rem, and are not physically detectable because of their virtual nature.

14.10 Mathematical Intuition and the Logic of the Internal Observer

It is by logic that we prove, but
by intuition that we discover

Henri Poincare

As we have seen, quantum space-time is a quantum computer that quantum-evaluates recursive functions that are the laws of physics in their most primordial and symbolic form. In agreement with Deutsch [15], we believe that the laws of physics determine which functions can be computed by a universal computer. Further, we claim that the laws of physics *are* the recursive functions that are quantum evaluated by space-time at the Planck scale [58].

The global properties of the recursive functions that are quantum evaluated by quantum space-time (fuzzy black holes-quantum computers, and the universe as a whole) are to be considered as the "universals in the thing". The same quantum computation performed by the brain microtubules, are to be considered as the "universals after the thing". This is why our minds are compatible with computable functions (and thus with the laws of physics). In fact, Deutsch says that, since any computational task that is repeatable or checkable may be regarded as the simulation of one physical process by another, all computer programs may be regarded as symbolic representations of some of the laws of physics. But it might be that not all the computational tasks performed by our brains are repeatable and checkable, in fact Penrose believes that most probably our thought is not algorithmic [36, 37]. Penrose might be right in saying that (some aspects of) consciousness are noncomputable, although some further considerations should be added to his statement, as follows.

If our mind is a quantum computer, we know that a quantum computer has the same computational power as a classical computer, (and this means that a quantum computer can compute only Turing-computable functions) although it is much more efficient. However, there are indeed some aspects of our consciousness that are not algorithmic in the usual sense. In a recent study [3], we considered the internal logic of a quantum computer, and found out that it is not the standard quantum logic endowed by an external observer, instead it is a paraconsistent, symmetric logic like basic logic [40]. In fact, we argue that the logic underlying quantum computation at the Planck scale is not just standard quantum logic, (as in the case of ordinary quantum computers), but it is basic logic.

In the internal logic of a quantum computer, there are two very strong axioms, the reverse of both the noncontradiction and the excluded middle principles, which are obtained as reversible measurements. These two strong axioms, which are the manifestation of a great amount of quantum information, as the superposed state is maintained, are associated with a very weak calculus (the conclusions are almost similar to the axioms, but the axioms are very strong). The weakness of the sequent calculus indicates that there is almost no algorithm, but the conclusions are not trivial, as the premises are very strong. This might describe the immediacy of mathematical intuition, once the mind is regarded as an internal observer of the whole quantum universe, as in this case the logic it is using is paraconsistent.

It is generally assumed that there is a fundamental difference between the "axiomatic reasoning" and the informal mathematical reasoning. Instead, we believe that there are no proofs in mathematics that can be obtained without following the usual path from premises to conclusions, the only difference, in the case of intuition, is that the path is much shorter than usual. Moreover, since the information stored in the axioms cannot be provided twice as the source cannot be duplicated because of the no-cloning theorem [50], intuition is given only once. In this sense we meet Penrose, as repeatability is of course an intrinsic feature of algorithms. The absence of repeatability is a property of basic logic, due to the absence of the contraction rule, by which an operation cannot be repeated once the context within a particular sequent has been exhausted.

As we have seen, it is the attitude of a mind to place itself as an internal observer with respect to the quantum universe as a whole that allows it to acquire mathematical intuition. In other words, mathematical intuition is an interactive task between the mind and the universe, in some sense different from the understanding of physical laws, which includes all minds and black holes. Mathematical intuition is a private communication between one single mind and the whole universe. This sounds much like Platonism, but in our case the Platonic world of ideas is replaced by the physical universe. Goedel's first incompleteness theorem [20], which somehow stresses the imperfectness of the Platonic world, in this context actually reveals the incompleteness of any possible unified physical theory. Intuitionistic logic, for which the law of excluded middle is invalid, replaces Platonism by a constructive approach to mathematics. Surprisingly enough, we see that paraconsistent logic, for which both the laws of noncontradiction and of excluded middle are invalidated, still leaves room for Platonism to a certain degree.

14.11 The Self

Know yourself

Socrates

14.11.1 The Self and the Mirror Measurement

In two recent papers [3, 56], we considered a new kind of quantum measurement, performed by a (fictitious) internal observer placed inside a quantum computer on a quantum-space background. This measurement is reversible because it is achieved by means of a unitary operator instead of a projector operator. Thus, the superposition can be recovered, and there is no loss of quantum information. Let us consider, for example the superposed state of one qubit: $|q\rangle = a|0\rangle + b|1\rangle$, where a and b are complex numbers, called the probability amplitudes, and the probabilities sum up to one: $|a|^2 + |b|^2 = 1$. Among all unitary 2×2 matrices acting on the qubit state as reversible measurements, there is one set of diagonal matrices, that we called "mirror" measurement:

$$M = \mathrm{e}^{\mathrm{i}\phi} \begin{pmatrix} \alpha & 0 \\ 0 & \alpha^* \end{pmatrix},$$

which has the property of leaving the probabilities unchanged, although modifying the probability amplitudes. The logical consequences of the mirror measurement, is that the internal observer gets rid of the noncontradiction principle. By symmetry, he also gets rid of the excluded middle principle. The internal observer is then endowed with a paraconsistent and symmetric logic, as we already said in the previous section.

There are two very strong axioms, in this logic, the mirror measurement leading to the one that states that: A and not A is true, which is the converse of the noncontradiction principle. The philosophical meaning of this axiom is that the superposed state $|q\rangle$ reflects in a slightly deformed mirror, that is, the diagonal unitary operator M, which just changes the probability amplitudes, but leaves unchanged the truth values (the identity operator being the perfect mirror). This analogy would suggest that the qubit has undergone a reversible self-measurement (without decoherence). The act of "looking at itself in the mirror" confirms the objective existence of the qubit. This is along the same line of thought as that of Mermin [35], who looks for an interpretation of quantum mechanics where objective reality should be separated from external observers and their knowledge.

We believe that the sense of "self" in human minds arises when a superposed state of tubulins undergoes a mirror measurement, without OR. This might sound like the antithesis of consciousness, as any conscious event originates from OR, which is subsequent to a quantum superposition. And in fact, self-awareness should be the purest form of consciousness! The sense of "self", however, seems to baffle our beliefs. This is due to the fact that in a mirror measurement, there is the maximal conservation of quantum information. And this maximal information is somehow stored and reused several times, without being ever dispersed in the environment. The sense of "self" in fact must be the most inner, persistent, indestructible feature of our mind.

14.11.2 Nonself

This is not mine, this I am not, this is not my Self

Buddha Gautama (563 B. C.)

One might wonder what happens when, instead of undergoing a mirror measurement, a quantum state of superposed tubulins is processed by a Liar measurement [3, 2]:

$$L = \mathrm{e}^{\mathrm{i}\phi} \begin{pmatrix} 0 & \beta \\ -\beta^* & 0 \end{pmatrix},$$

which is an off-diagonal unitary matrix, or by a general unitary quantum logic gate [3, 56]:

$$U = \mathrm{e}^{\mathrm{i}\phi} \begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix}.$$

In the first case, the probabilities (and the truth values) are interchanged, in the second case, they are mixed up, although the state is still superposed, until it decoheres by OR.

The mirrored self is a very peculiar case, most of the times our self is reversed, or distorted, and then, annihilated. A conscious experience arises after OR. We give up our self every time we become conscious about something else. Then the process restarts again, and again.

We recover our self, and then we lose it once more.

In summary, a conscious experience coincides with the minimum quantum information about itself of a quantum system, while the self corresponds to the maximum quantum information.

14.11.3 The Universal Self: The Universe and the Mirror

The objective world simply is; it does not happen.

Herman Weyl

The universe, as a whole, can undergo, as any other quantum system, a mirror measurement. However, the consequences of this cosmic mirroring are by far the most intriguing. In fact, the universe is the ensemble of all existing things, and its mirroring is the mirroring of everything that exists at the same time. The objective existence of existence itself is recognized. All things then acquire a collective sense of self, as if they were a Bose–Einstein condensate [5].

14.11.4 The Universal Self: The Mathematical Truth

Why are you a physicist?
Why aren't you a mathematician?

Paul Erdos

I agree with Deutsch [14], who recognizes the dependence of our mathematical knowledge on physics. Mathematics is in fact the language by which we express our knowledge of the physical world. The effectiveness of this language is due to the fact that the laws of physics are Turing-computable functions. However, like Deutsch, I also believe that our knowledge of the mathematical truth (or mathematical intuition) does not depend on physics.

As a physicist, I feel a kind of reverence for mathematics. Most probably, this leads me to idealize it too much, and all my considerations about mathematics sound quite platonic. "This is not fine", a colleague of mine told me, who is a logician: "The world of ideas does not exist, we are the ones who construct Math". Although respecting intuitionist philosophy, I suggested the following compromise.

When the quantum universe, mirrors itself in a mirror quantum logic gate, its superposed state gets slightly changed, but maintains the truth values at the "right place". This, as we have seen, is the universal self. Is not that a kind of universal (although slightly imperfect) mathematical truth? Actually Brouwer, the founder of intuitionism, considered the self closely related to the "immanent truth" [7].

Also, for Brouwer, "The ego (self), at the onset of mathematical activity, is simply given; introspection is its natural form of knowing... the ego is... consciousness transformed to mind. The primordial intuition..is direct insight, introspection by and in the individual mind..." [45].

Similarly, we would argue, the universal self might be related to a "universal immanent truth", which is recognized, by the observer, as the universal mathematical truth. In other words, when the human mind places itself as an internal observer of the universe as a whole, (the master program), it behaves as a fixed point, and is able to grasp the code (the mathematical truth).

14.12 Conclusion

The theory of Knowledge
is a Product of Doubt

Bertrand Russel

In this chapter, we described the early inflationary universe as an ensemble of quantum gravity registers in parallel. At the end of inflation, the superposed state self-reduces by reaching the quantum gravity threshold as in Penrose's objective reduction model. This self-reduction can be interpreted as a primordial conscious experience. Actually, the number of quantum gravity registers involved in the OR equals the number of superposed tubulins in our brain, which are involved in the Orch OR, leading to a conscious experience. It should be noticed that, in this model, the quantum gravity registers in parallel are parallel universes. This interpretation is very much along the same line with Deutsch' idea relating quantum computers to parallel universes (the "multiverse") [16].

However, at the end of inflation, only one universe is selected, the one that is endowed with that particular amount of entropy that makes it our world. Further, the qubits of the selected quantum gravity register get entangled with the emergent environment (radiation-dominated universe) and collapse to classical bits.

We make the conjecture that the postinflationary universe starts to organize itself, very likely as a cellular automaton, and necessarily produces self-similar computing systems. Actually, the CA-universe can undergo two different configurations, a classical one, performing classical computation, and a quantum one, performing quantum computation. The same can be done by tubulins in our brain. The universe, as a quantum computer, has subroutines that are black holes quantum computers with fuzzy spheres as event horizons. These black holes, and the universe itself as a whole, quantum evaluate the laws of physics in their primordial form. The outputs are the universals. Due to quantum entanglement, our minds operate in parallel with black holes in the computation of the physical laws, but the outputs are interpreted in a subjective way. The results of these computational tasks, are our concepts on the universals and this is an aspect of our mind that, although being quantum, is certainly algorithmic. It is in fact the algorithmic nature of such aspects of our consciousness that allows us to comprehend the physical world. Qualia might be considered units of the universals.

However, there are two aspects of our consciousness that are not algorithmic in the usual sense. They are mathematical intuition, and the self. The first corresponds to the attitude of one single mind to place itself as an internal observer, endowed with paraconsistent logic, with respect to the quantum universe as a whole. The second corresponds to the reversible self-measurement of the mind by means of the mirror-quantum logic gate. The self-measurement of the whole quantum universe in the mirror-gate corresponds to the universal self, which we interpret as the primordial origin of any form of self-awareness in terms of mathematical truth.

To conclude, we would like to stress the fact that, in this work, our study of consciousness required the use of some theoretical physics, some mathematics and also some philosophy. We think, in fact that consciousness is a highly interdisciplinary issue, and deserves the best of combined efforts from different disciplines. In this field, the "expert" does not exist yet. However, as Niels Bohr said once, "An expert is a man who has made all the mistakes, which can be made, in a very narrow field".

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