A Mind/Mathematics Dualistic Foundation of Physical Reality

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Abstract. In a refined version of Wigner's interpretation of quantum physics, the Universe is explained as a part of the mathematical world (a specific history inside Everett's many-worlds) that is distinguished by the event of being consciously perceived. Physics focuses on the mathematical side of this combination, that is a Platonic mathematical realm slightly less than infinite. Consciousness provides the substance of time and randomness (beyond their mathematical forms as 4th dimension and probability laws).

Non-mathematical worlds

The world displayed by Physics is *highly mathematical*. But how else can a world be ? Here are some alternatives:

- A *lowly mathematical world* is an algorithmic world, such as Conway's Game of Life. Its laws are very basic ; all their consequences come by brute computation with a fixed method that imaginative tips cannot much help to shortcut. While well-designed configurations can simulate any Turing machine, most others behave chaotically. Even thousands of known cases may not help to guess the effect of another pixel of disturbance, that seems as boringly random as a list of prime numbers. The repeated application of the same evolution law comes each time “without a reason”.
- At the other extreme, a *non-mathematical world* is purely made of feelings and qualities, where no quantitative measure can objectively be applied.

But if a large amount of quantitative data is measured, it can be analyzed in terms of its correlations, i.e. its global probability law, which determines a compression format up to size-preserving conversion : the compressed file expresses the contingent part of reality causing or explaining the file of observed data, while the compression format expresses the law (how things work ; if its success breaks when the range of observations expands, the variations of the law are then contingent data to add to the compressed file).

This quest may face obstacles:

- For a law to be verified, it needs to effectively compress the file of observations, i.e. that observed data are broad enough for parts of them to exceed the complexity of their hidden causes (the corresponding part of the compressed file). But for example, fluctuations in stock prices cannot reveal themselves the more complex economic causes of their jumps behind their Brownian appearance.
- As the range of observations expands and more complex laws are tested, the quest might never converge to a “final best law”. But what can make a world follow an infinite series of more and more accurate and complex laws that “just happen” without following themselves any deeper law ?
- If the most compressed version of the file we found is still heavy, its content looks random, i.e. unexplained, but the impossibility to better explain (compress) this file, remains anyway unprovable (Chaitin's theorem).
- The true law may escape this search if it is not algorithmic, may it be still mathematical (e.g. Chaitin's constant), or not mathematical at all.

But how can a non-mathematical law be conceived ?

Psychological laws and the Turing Test

Pieces of art such as musical compositions, follow “artistic laws”. Similarly, psychology has its laws : people are more likely to follow “meaningful” behaviors, than “absurd” ones. This law is non-algorithmic if the measure of “meaningfulness” cannot be algorithmically defined. The reality of psychological laws that do not admit good algorithmic approximations, can be verified by the Turing Test : candidate algorithms are found to fail approaching psychological laws when their responses are humanly assessed as “absurd”, while those produced by humans are found “meaningful”.

Instead of algorithms, we cannot use any exhaustive list of “meaningful behaviors” because, to be ready for any challenge, this list (too big to be stored in a computer) requires real minds to be produced. The feeling that responses came from a real person, would no more be a complete lie. It would be a partial lie if the real author was a comedian instead of the claimed person, but his simulation may fail to sustain realism in the long term unless he somehow really lived what he is pretending.

Time and unpredictability in mathematics

Such an unpredictability of behaviors that require “real substance” to provide a correct result, happens in mathematics too. A typical example is the undecidability of the halting problem: no computation can reliably predict in any unlimited but finite time (using potential infinity) the last word of possible outcomes of any other computation with unlimited resources (as the claim that an algorithm will never stop depends on actual infinity). Similar things happen “after infinity”: when interpreting a theory in any possibly infinite but fixed model, the interpretation process of formulas is time-ordered from the simplest formulas to the most complex ones whose values depend on those of their sub-formulas. By the Truth Undefinability theorem, the general definition of truth for all formulas with variables in a given set, cannot be written by a single formula with variables in the same set, but requires the use of a bigger set. This can be understood in terms of a time order of mathematical reality (independent of our time): the bigger set is the future set of the past encompassing the current past with the infinity of its possible present formal descriptions.

Mathematical vs. conscious existence

All possible computations mathematically exist (or will exist) as mathematical systems. What can a “physical existence” of a world where a computation “happens”, bring to it above other computations ? All their elementary steps anyway repeatedly happen many times in any “physically existing” or “non-existing” universe. A specific universe has a specific series of operations “happening together at a place” with the mathematical property of representing a specific global computation, but then what ? How could a melody exist, not just as a succession of sounds but indeed as a melody, without somebody to hear it ? How can a thought exist, not just as a mathematical property of brain computation but as feeling something, without the fundamental addition of an immaterial soul inside the brain to actually feel what the brain is computing ?

No concept of “physical existence” given to a universe “on a fundamental level”, can add anything to its emergent (non-fundamental) mathematical structures of brain computation to make them “exist” any more than similar structures “happening” in physically non-existing (but mathematically existing) universes. As we shall see, the conscious perception of mathematical structures can explain and constitute their “physical existence", instead of the other way round.

Consciousness can explore mathematics, but mathematics cannot describe consciousness. While mathematical reality is analytic (systems are divisible into parts, down to mute elements), consciousness is fundamentally synthetic (its divisions can only be approximations). Conscious events are subject to time order, which is their order of relative existence: an event A "coming before" an event B is an event that exists inside B (in memory, even if it may be hard to retrieve). In other words, past events exist but future events are not determined yet.

Consciousness happens to be approximately split as a multitude of individual minds, that coexist "somewhere deeply inside" each other, in a common Matrix (God), like individual physical objects may be said to coexist in a common physical space from which they cannot be dissociated.

A physical universe needs a "probability law"...

A purely mathematical world with deterministic laws (with a limited size of compressed contingent data no matter the volume of observations, like the Mandelbrot set that only depends on where you look) would fail to include free will (expression of psychological laws). It could be consciously observed, but not modified. This would not be any hospitable world. In a world of pure feelings we cannot mathematically analyze anything. Remaining possibilities are worlds where observables may either (from the least to the most mathematical world):

- Take arbitrary values only subject to psychological laws, where any approximation by mathematical laws would be unsatisfactory;
- Be subject to a mathematically defined set of possibilities, among which choices are ruled by psychological laws (as in the previous case);
- Follow a definitely favorite mathematical probability law (thus excluding outcomes whose probability cancel) from which actual outcomes may deviate when psychological preferences apply.

One might say, "the latter case is nonsense, what about admitting a fully respected mathematical probability law ?". But *that* would be nonsense : as physical existence is an act of consciousness towards some mathematical structures, there is no other source of randomness but combinations of mathematical necessities with sorts of conscious choices.

Unless a result is already settled, all what a "probability law" rigorously does is to exclude zero-probability cases from the range of possibilities; all other cases remain possible by definition. The concept of a deviation, among them, from a physical "probability law" towards a psychological preference, is a psychological concept with no natural mathematical formalization in terms of which the law can be said to be "physically broken". According to some results of parapsychology experiments, people seem able to influence the outcomes of quantum random generators they observe.

Still the probability law remains meaningful, not only by the trends it gives from which conscious choices have a "difficulty" to depart, but also by the physical presence of many undetermined processes to which no psychological preferences would happen to provide any trend : mathematical probabilities need to be provided as "default behaviors" ready to apply in such cases.

...about mysterious "observables"

Mathematical theories can only define probabilities as numbers. Effective randomness is the practical property for a piece of data to come from a source (cause) that happens to be sufficiently independent of some specific concern, to not be systematically biased towards it. Metaphysical randomness is an act of consciousness choosing only one possibility to become real, while alternatives had physical chances of being picked up instead ; it is effectively random when done independently (typically, by the Matrix instead of the concerned mind).

No mathematical law can describe such conscious acts nor force them to happen in any specific way, time or place. The transition from mathematical probabilities to actual randomness, turning "undetermined things happening" into shapes of clear appearance, will have to refer to "measurements" of "observables" arbitrarily introduced from the outside of physics.

Trying to strictly follow the predictions of the rest of mathematical laws while dismissing these mysterious measurements (as what actually "happens when nobody is looking"), the diverse possibilities would seem to keep coexisting in parallel, weighted by their "probabilities". Expectedly, when trying to explain this occurrence of randomness from additional physical laws without the fundamental introduction of conscious observers, attempts to see measurement results as predetermined would run into many problems, while ideas of later random choices would mysteriously need them to successfully happen no later than when someone looks (as if it ever mattered).

In short, the expectable laws of physics would look pretty much like those of quantum mechanics. Welcome home. ;-)

Mind makes collapse interpretation

To create the Universe, Consciousness first chose a mathematical law as "theory of everything" of physics: that is a theory of a Hilbert space with more structures (or the like, quantum gravity will tell). It defines Everett's many-worlds landscape, that is the landscape of "all possible physical worlds" with the same law, but at first, all with only the same mathematical existence they had (just like mathematics is the science of all possible mathematical worlds, which all mathematically exist).

During conscious time, specific worlds in this landscape may receive "physical existence", that is, the occurrence of being "physically perceived" by consciousness. The physical Universe is the trajectory of this exploration of the Hilbert space by consciousness : at every conscious time, the physical state (density operator, nicknamed "wavefunction"), is the projected mathematical image in the Hilbert space, operated by the Matrix, of the heritage (universal conscious memory) of all past physical perceptions. By this computation, the Matrix obliges all physical perceptions to stay "mathematically coherent" with each other inside the Hilbert space.

The asymmetry of conscious time gives the thermodynamical time orientation, letting any physical state be only and entirely given from the past (only past perceptions exist, and the state of the Big Bang had to be completely specified, probably as it was exhaustively perceived too). This orients decoherence (the process of "measurement by a physical device"), which provides the next possible observables with their classical probability laws. The "wavefunction collapse" is the *metaphysical process* of adding a new physical perception (of a possible world after decoherence) to the heritage of past perceptions which determines the physical state. Contrary to ideas of quantum consciousness by

Penrose and others, arguments can be found for expecting observations to only take place after decoherence (thus, the action of free will in the brain would not need any entanglement of quantum states). As decoherence can only be fuzzily defined as an emergent property, this condition fits well the non-physical nature of the collapse.

The mathematical realm involved in Physics

Quantum theory has remarkable properties. One of them is its mixture of computability and continuity.

On the one hand, it has holistic, Platonic properties giving its internal processes a character of necessity. The physical state at a time is both continuously related and inseparable with that of other times: a state "evolves" as it does because of its nature (the necessity of its internal structure) rather than by an arbitrary computation rule. Physical "times" are not clearly separate from each other but relative to an arbitrary, unphysical choice of divisions of space-time into slices (relativity of simultaneity). Quantum field theory is naturally expressed by fixing the Hilbert space and the state in it, and expressing physical events in space-time as variable operators depending on their locations.

But it is also computable. The implicit infinity of infinitesimals in its continuous variations, does not behave as an actual infinity but a potential one only. Like Euclidean geometry that is algorithmically decidable and unlike arithmetic, results can be computed (though we currently face computational divergences, whose resolution would need reformulations, maybe from quantum gravity).

But these computations must be processed in a different order than the physical time order. Physical time (a geometrical order that consciousness will follow, to "embody" the conscious time) is clearly disconnected from computational times, as can be seen from its reversibility at the fundamental level of the Hilbert space (ignoring thermodynamics and wavefunction collapse), and from the very kind of mathematical formalism in which Physics is expressed. Indeed, Feynman diagrams, representing histories of particle interactions across space-time, constitute tensorial expressions. Unlike "ordinary" mathematical expressions, whose tree-like structure orders the evaluation of their symbols from the branches (sub-expressions, that would be "past") to the root (the main symbol giving the final result), tensorial expressions need not have a tree structure, and their interpretation can be equivalently processed in any order.

While no exact prediction can be reached in any finite number of operations, after some computational time (depending on the size of the physical system), the next digits of results only improve the accuracy of probabilities of observable outcomes. But to play their roles of probabilities, these numbers do not need to "actually exist" with infinite accuracy, they only need to be *divinely guessed*. Thus, Physics uses a very Platonic part of mathematics, involving the infinite meaning of theories beyond finite computations, but still not affected by the formal uncertainties on actual infinity revealed by Tarski's Truth Undefinability and Gödel's Incompleteness theorems.

The multiply simple structure of quantum theory

Mathematics is characterized as a self-contained study of logical necessities on clearly specified systems; when facing multiple possibilities, it admits all of them as equally valid in parallel. Other sciences may depart from this in several ways : dealing with what cannot be rigorously specified (psychological laws, economics); what is determined but computationally too complex to be humanly deduced from fundamental laws, and thus needs experiments to fill the gaps of understanding (chemistry, materials physics); or what is relative to a large number of uncontrolled contingencies and possible unknown extra laws, for which observational input is clearly needed (biology, astronomy).

Physics focuses on mathematical aspects of the world, i.e. where any undefined, too complex or contingent factor is either controlled, simplified or ignored by taking “the general case”. This usually happens in studies of planetary systems, whose evolution laws are satisfyingly determined by assuming that planets are roughly spherical and do not spontaneously explode, though no fundamental law absolutely obliges this.

Quantum theory makes the universe remarkably mathematical, as the mathematical roots of its laws are not only found at one level of process with one specific law, but at continuously many levels.

Possible states of quantum systems can be locally analyzed as ranging over (continuous quantum superpositions, or probabilistic combinations, of) a finite number of possibilities depending on the size and available energy of the system. This way, any local effect (a later state of a system, which defines its probabilities of measurement results) only depends on a finite “number of causes” (amount of quantum information), those of the physical state of what is there at a previous time in the past light cone of the effect. All laws and observables can be described as matrices of quantities relating those states.

Things are mathematical when this list of possible states is not too big, or not too complex. But this condition can be achieved in many ways: either by looking at small enough scales to find only few possible states with energies comparable to the average available pack of energy, or looking at wider systems but with a low enough temperature so that only few global states (of entanglement between states of their smaller components) will have low enough energy for significantly contributing at this temperature, according to the Boltzmann distribution.

Still, things can also be mathematical in another way at macroscopic scales with high temperatures (especially above 1,000 K or the like), as the larger number of available states provides bridges mixing them all, quickly dissolving any specific state into the uniform Boltzmann distribution determined by the temperature.

Physics also involves remarkable mathematical concepts, where one theory (law) admits several equivalent worthy formalizations and computation methods. For example, quantum fields display both aspects of waves and particles without contradiction. This contributes to infirm any idea of a specific “physical cause” behind the mathematically defined probability law on observables. God looks like a great mathematician.

Complexity and life

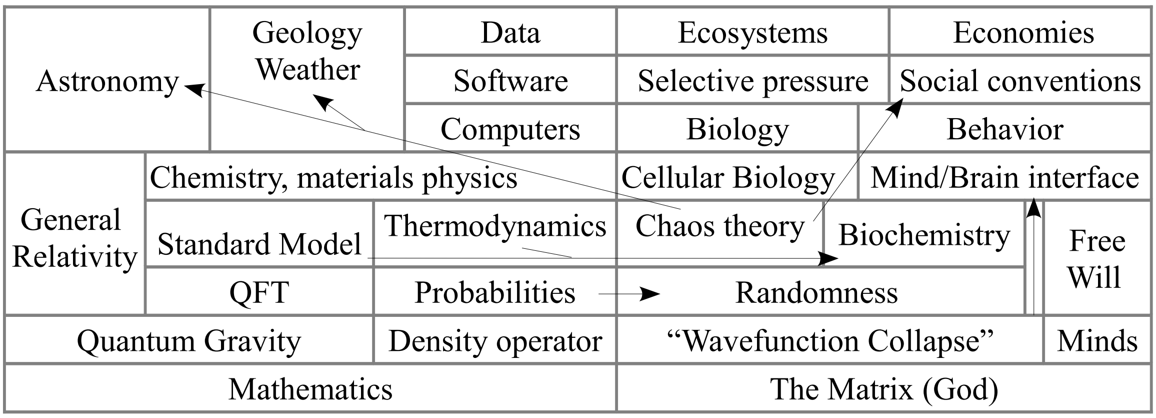
However, God does not look like a great humanist. While the Universe could always be contemplated by free souls for its mathematical beauty, it took a lot of time since the Big Bang, and a very special combination of factors, to produce the kind of physical environment that is suitable for life as we know it, i.e. for the development of complexity, where conscious choices could produce interesting effects not just mathematically but also psychologically, in a more stable and meaningful way than in the butterfly effect.

First, it needs a place at the right temperatures, chemical composition and flow of energy (opportunity of entropy creation), for molecules to undergo various reactions to evolve in some ways but not in any ways, to not lose all their information.

Then, the emergence of efficient bodies for properly hosting souls, able to thrive in an unfriendly environment and giving free will a wide range of possible actions, took billions of years of chaotic evolutionary history of trials and errors from biochemistry to cells and more complex organisms.

Complex phenomena are roughly structured as a foundational hierarchy, with different conceptual levels corresponding to the different scales at which things form "distinct objects", and/or to the relation of foundations (general laws) vs. contingent data. Objects at each level have their own laws of behavior, so that their interactions with contingently found peer objects, form (contingent) organizational structures that constitute the law of the next level.

Possible laws range from the simplest, chaotic (disorganized) ones such as the "jungle law" (selective pressure), where objects come in bulk and are best described by probabilities and averages, to the most complex (organized) ones, whose details may depend on some persisting contingent data : bodily functions depend on DNA information ; economic and social systems also depend on educational heritage, available technologies and political and monetary conventions.



The foundational hierarchy of the Universe

The Problem of Evil ?

There is a big paradox: the world is not as hospitable as it might be, so we ask, "My God Why" ? It is very strange indeed that psychological laws (free will) only physically operate at the level of individual minds in their respective incarnations, without noticeable coordinated action either on the larger scale (telepathy, divine guidance) or the smaller one (intelligent design on DNA mutations), letting evolution take place at the slow, wasteful rhythm of Darwinian selection. Some of these facts are connected: Darwinian "rules" both explain and require embodied souls to ignore their immaterial nature and adopt a relative selfishness, giving intelligent forms of soul embodiment and bodily abilities their natural selective advantage.

But, maybe even more strange, is that we usually do not complain about

- The suboptimal design of our bodies (risks of failure and illnesses)
- The relative deficiencies of most people's brain, making school learning so hard
- How the brain makes pain and troubles "painful" to the mind
- How is love sometimes too selective to work out
- The worse fate and handicaps of animals in nature, and of all our and their ancestors since the emergence of life
- The worse injustice suffered by animals in intensive farming and laboratories

Instead, we usually complain about poverty, injustice and tyranny, which we attribute to a lack of divine hand on the Universe.

There's Plenty of Room at the Bottom ![1]

And what makes this focus of complaints strange, is that these troubles are actually those we have the easiest opportunities to fix by ourselves, as we already started doing by science, technology, diverse organizations, political systems, and online social networks, in a ridiculous amount of time compared to the history of life on Earth.

Or rather, it would be easiest... if only we cared about it, instead of wasting our works and dreams elsewhere as we do:

- Religions deny the possibility for society to be anything more than the disorganized sum of its parts, focusing instead on either blaming human nature for society's ills, and/or accepting them as God's will.
- Science fiction focused its dreams on Artificial Intelligence replacing human minds, interstellar travels and other technological gadgets. Only few authors dreamed about the Internet, but who expected online social networks to assist democratic expressions and political revolutions ?
- Economists care to describe the nonsense in which we are, keeping their ideas as faithful to it as they can.
- Mathematicians and other hard creative thinkers are expected to either stick their thoughts to pointless abstract foundations or to create new gadgets at the service of current institutions, as they would be ignored or blamed for their abstraction and “complication” if they dared to get into matters of public interest.

Should we explain what technology is ? Technological progress is the activity of *reprogramming the mathematical structure of effective realities*, from previously chaotic to more versatile features. Thus, as soon as we could restructure physical objects into powerful Universal Turing Machines (computers) and connect them worldwide (Internet), *any algorithmically expressible law of external reality* can be built on top of this as soon as we can invent it and express it as new software.

Thus, all we still need is a combination of highly theoretical work and software development, not to understand our economic and political systems as they are, but to redesign them as they should be to best connect their given basic elements : humans as they are, given their list of basic needs, which better online social networks should be able to fulfill : to learn, find hosting, events, carpooling, love, jobs (which are activities of processing data better than known algorithms to fit human needs), make online transactions (where money is a social convention in need of logical redesign), filter truth and reputation from errors or propaganda, and form better political and judicial systems.

I described the sketch of such a new social network, but could not find anyone else to care understanding it and working on it. People prefer to think small, and about something else. But if not even a few people care to think deeply enough to develop the exactly right design of complex solutions to the needs, then we keep bad solutions such as corrupt political systems and wasteful academic systems, while more ill-designed solutions can come and take over worldwide popularity by surprise, despite all the risks of failures and abuses hidden behind their visible advantages.

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1 title of a famous lecture by R. Feynman in 1959 anticipating the development of nanotechnologies

References

(All these links are already in the main text)

By the present author

Introduction to the foundations of mathematics
http://settheory.net/foundations/introduction

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How mathematical theories develop, and admit equivalent formalizations
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Short presentations of the Truth Undefinability and Incompleteness theorems
http://settheory.net/incompleteness

Interpretations of quantum physics :

- Introduction : http://settheory.net/quantum-interpretations
- Problems with Bohmian mechanics : http://settheory.net/Bohm
- The Many-worlds, and a note on the Big Bang : http://settheory.net/many-worlds
- Mind makes collapse interpretation : http://settheory.net/quantum-mind-collapse

On Platonism and the demarcation of Mathematics
http://settheory.net/foundations/metamathematics

The demarcation of Physics, and the hierarchy of physics theories
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